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Quantum Dynamics of A Bulk-Boundary System

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Abstract

The quantum dynamics of a bulk-boundary theory is closely examined by the use of the background field method. As an example we take the Mirabelli-Peskin model, which is composed of 5D super Yang-Mills (bulk) and 4D Wess-Zumino (boundary). *Singular interaction* terms play an important role of canceling the divergences coming from the KK-mode sum. Some *new regularization* of the momentum integral is proposed. An interesting background configuration of scalar fields is found. It is a localized solution of the field equation. In this process of the *vacuum search*, we present a new treatment of the vacuum with respect to the *extra coordinate*. The "supersymmetric" effective potential is obtained at the 1-loop full (w.r.t. the coupling) level. This is the bulk-boundary generalization of the Coleman-Weinberg's case. Renormalization group analysis is done and the correct 4D result is reproduced. The *Casimir energy* is calculated and is compared with the case of the Kaluza-Klein model.

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1 Introduction

Through recent several years of development, it looks that the higher-dimensional approach has obtained the citizenship as an important building tool in constructing the unified theories. It appears with some names such as "Randall-Sundrum model", "brane world", "extra-dimension model", "orbifold model", etc. Before the appearance of the new approach, supersymmetry (SUSY) was the main promising tool to go beyond the standard model. Among many ideas in the higher-dimensional approach, the system of *bulk and boundary* theories becomes a fascinating model of the unification. A boundary is regarded as our world. It is inspired by the M, string and D-brane theories[1]. One pioneering paper, which concretely describes the model, is that by Mirabelli and Peskin[6]. They take the 5D supersymmetric Yang-Mills theory as a bulk theory and make it couple with a boundary matter. The boundary couplings (with the bulk world) are uniquely fixed by the SUSY requirement. They demonstrated some consistency in the *bulk* quantum theory by calculating *self-energy* of the scalar matter field. Here we examine the *effective potential* and the *vacuum energy* of this system. We investigate further closely the role of the bulk fields and the singular interactions.

A field theoretical analysis of the bulk and boundary system was recently done in the work by Goldberger and Wise[2]. They try to tackle the problem by the "generalized" use of the renormalization group. Randall and Schwarts[3, 4] also attacked the same problem by introducing a special regularization of the ultraviolet divergences guided by the idea of the holography. We take a different approach to the bulk and boundary system. (We will see, however, some similar results.) It is, at present, hard to show any consistency (such as renormalizability, unitarity, etc.) in the higher dimensional quantum field theory. It is, at least perturbatively, *unrenormalizable*. We would rather regard the bulk world as an *external heat-reservoir* which gives some "freedom" to the boundary world and "define" or "regularize" the 4D dynamics. The external world of the bulk *classically* and *quantumly* affect our world of 4D, and vice versa. In this circumstance we focus on the renormalization properties of the 4D world. We examine a way to treat the linear (power) divergences coming from the bulk quantum effect. Another important aspect is the present treatment of the extra axis. We will find some "freedom" in the definition of the vacuum. Z^2 -symmetry plays an important role there. We can naturally introduce the singular behaviour for some scalars. One important merit of the bulk-boundary approach is that the anomaly phenomenon (of the 4D world) is naturally accepted as a current flow which goes out through the wall or comes into the 4D world.[5].

Contrary to the motivation of the original work of ref.[6], we do *not* seek the SUSY breaking mechanism, rather we keep the supersymmetry and make use of the SUSY invariant properties in order to make the analysis as simple as possible. The SUSY symmetry is so restrictive that we only need to calculate some small portion of all possible diagrams.

As the analysis of the effective potential of the 5D model, we recall that of the Kaluza-Klein(KK) model[7]. The dynamics quantumly produces the ef-

fective potential which describes the Casimir effect. The situation, however, is contrastively different from the present case in some points.

1) The present approach realizes the 4D reduction by the localized (along the extra axis) configuration (kink, soliton, delta-function), whereas KK does it by the shrinkage of the radius of the extra S_1 space.

2) KK does not use Z_2 -symmetry whereas the present one exploits it in order to make a singular structure at $x^5 = 0, l$ (fixed points) where the 4D worlds are. The discrete symmetry imposes a nontrivial boundary condition on the vacuum.

3) KK takes the condition scalar field = constant in order to find the vacuum configuration, whereas, in the present case, we do the *new* treatment of the vacuum by allowing the extra-coordinate dependence on some scalars.

4) The present model is supersymmetric, whereas KK is not.

5) In KK, the scalar field comes from the (5,5)-component of the 5D metric (and partially from the dilaton). The 5D quantum effect produces the effective potential which can be interpreted as the Casimir force induced by the vacuum polarization between the l separated objects. On the other hand, in the present case, the scalar components come from various places: the 5th component of the bulk vector, the bulk scalar and the boundary scalar fields. Hence the vacuum structure becomes much richer.

6) The present model has, as the characteristic length scales, the thickness parameter (brane tension) besides the period of the extra space. As the vacuum energy calculation, we should see the dependence on both lengths.

The present model shares common properties with those of the RS-model in the points such as localization, Z_2 -symmetry, bulk-boundary relation, etc. (The comparative aspect of the KK model and the RS-model is explained in ref.[8].) We could regard the present result about the Casimir energy as some RS counter-part of the result obtained by Appelquist and Chodos for the case of the KK-model.

The concrete object we will obtain is the effective potential. The formalism itself is very orthodox. The new point is its application to the 5D bulk-boundary system. The system is much extended from the ordinary field theory. The effective potential is well-established in the field theory. Especially in the middle of 70's much literature appeared. One of the famous outcome is the Coleman-E.Weinberg potential[9]. In the SUSY theories, Miller proposed a useful method, called AFTM[10], based on the tadpole diagram method by S. Weinberg[11]. It was applied to unified models[12]. We will take another formalism, the background field method, by B.S. DeWitt[13] and G. t'Hooft[14].³ The new formulation of the bulk-boundary system is another aim of this paper.

We summarize the new points as follows:

- (1) background-field formulation of the bulk-boundary theory,
- (2) $\delta(0)$ singularity problem is solved,

³The use of the background field method in the brane world analysis is stressed by Randall and Schwarz[3, 4]. They develop a perturbative treatment in the AdS₅ 5D bulk theory. They try to solve the similar problems to the present ones. Especially perturbative treatment, log versus power divergences, regularization, renormalization group running of the coupling. They do not use a SUSY theory. They focus on the bulk gauge field theory.

- (3) a new proposal for resolving the UV-divergences in the 5D quantum S^1/Z_2 orbifold theory,
- (4) new treatment of the vacuum in the presence of the extra-space,
- (5) Casimir energy calculation.

Some of the present results are briefly reported in ref.[15].

The paper is organized as follows. In Sec.2, we introduce the present formalism of the background field method, in the analysis of the effective potential. The simple model of Wess-Zumino is taken as an example. Here we explain the "supersymmetric" effective potential. Mirabelli-Peskin model is explained in Sec.3. It is a typical bulk-boundary model based on 5D SUSY. In Sec.4, we quantize the model using the background field method. A new treatment of the background field, in relation to the extra coordinate, is presented. This leads to an interesting background solution (vacuum) which describes the field-localization. Feynman rules are obtained for the perturbative analysis in Sec.5. The *singular* vertices, which involve the delta function, appear. Some Feynman diagrams are explicitly calculated. We take into account both bulk and boundary quantum effects. We will find that the singular interaction terms play the role of the "counter-terms" to cancel the divergences coming from the KK-mode sum. In Sec.6, the mass matrix appearing in the 1-loop Lagrangian is obtained. This is the preparation for the 1-loop full calculation of the next section. Assumption of the form of the background field about its extra-coordinate dependence is crucial for the present analysis. In Sec.7, the effective potential is obtained. Two typical cases, A and B, are considered. In Case A we look at the potential from the vanishing vacuum of the brane matter-field. The final form of the potential is similar to the 4D super QED. In the intermediate stage, we find a *new* type Casimir energy which is characteristic for the brane world. In Case B, we obtain the potential for the no brane configuration. In the intermediate stage, we find the ordinary type Casimir energy. The effective potential has rich structure. We conclude in Sec.8. We relegate some important detailed explanation to three appendices. App. A treats the super QED which is a good reference point in the analysis of the bulk-boundary theory in the text. App. B provides the calculation of the eigenvalues of the mass matrix of Sec.6. The results are used in Sec.7. App. C explains the concrete form of the present background fields. They satisfy the field equation with the required boundary condition.

2 Effective Potential of Wess-Zumino Model

In order to explain the background field approach to obtain the effective potential, we take the simplest 4D SUSY theory, that is, the Wess-Zumino model:

$$\begin{aligned} \mathcal{L}[\psi, A, F; \lambda, m] = & i\partial_m \bar{\psi} \bar{\sigma}^m \psi + \bar{A} \partial_m \partial^m A + \bar{F} F \\ & + [m(AF - \frac{1}{2}\psi\psi) + \frac{\lambda}{2}(AAF - \psi\psi A) + \text{h.c.}] \quad , \end{aligned} \quad (1)$$

where $(\eta^{mn}) = \text{diag}(-1, 1, 1, 1)$. The notation is basically the same as the textbook by Wess-Bagger[24]. ψ is a Majorana fermion, A is a complex scalar field and F is an (complex scalar) auxiliary field. The general background field method [13, 14, 16] tells us that the (DeWitt-Wilsonian) *effective action* $S^{eff}[\xi, a, f]$ is given by

$$\exp\{iS^{eff}[\xi, a, f]\} = \int \mathcal{D}\psi \mathcal{D}A \mathcal{D}F \times \exp i \int d^4x \left\{ \mathcal{L}[\xi + \psi, a + A, f + F] - \left. \frac{\delta \mathcal{L}}{\delta \Phi^I} \right|_b \Phi^I \right\} , \quad (2)$$

where $(\Phi^I) \equiv (\psi, A, F)$ are the *quantum* fields and their *background* fields $(\Phi^I)|_b \equiv (\xi, a, f)$. We define the *effective potential* V^{eff} as the *non-derivative* part of S^{eff} . A simple and practical way to pick up the part is to consider the case:

$$\xi = 0 , \quad a = \text{const.} , \quad f = \text{const.} , \quad (3)$$

where we put $\xi = 0$ from the requirement of the Lorentz invariance of the vacuum and "const." means a constant.

$$\begin{aligned} \exp\{-iV^{eff}[a, f]\} &= \exp i\{-V_0^{eff}\} \\ &\times \int \mathcal{D}\psi \mathcal{D}A \mathcal{D}F \exp i \int d^4x \left\{ \mathcal{L}_2 + \text{order of (quant. field)}^3 \right\} , \\ -V_0^{eff} &= \bar{f}f + (ma + \frac{\lambda}{2}a^2)f + (m\bar{a} + \frac{\lambda}{2}\bar{a}^2)\bar{f} \equiv \mathcal{L}_0 , \\ \left. \frac{\delta \mathcal{L}}{\delta \Phi^I} \right|_b \Phi^I &= (\bar{f} + ma + \frac{\lambda}{2}a^2)F + f\chi A + \text{h.c.} \equiv \mathcal{L}_1 , \quad \chi \equiv m + \lambda a , \\ \mathcal{L}_2 &= i\partial_m \bar{\psi} \bar{\sigma}^m \psi - \frac{1}{2}(\chi\psi\psi + \bar{\chi}\bar{\psi}\bar{\psi}) + \frac{1}{2}Q^\dagger M Q , \\ M &= \begin{pmatrix} \square & \lambda\bar{f} & 0 & \bar{\chi} \\ \lambda f & \square & \chi & 0 \\ 0 & \bar{\chi} & 1 & 0 \\ \chi & 0 & 0 & 1 \end{pmatrix} , \quad \square = \partial_m \partial^m , \end{aligned} \quad (4)$$

where the scalar quantum fields are denoted by the column matrix Q : $Q^T = (A, \bar{A}, F, \bar{F})$, $Q^\dagger = (\bar{A}, A, \bar{F}, F)$. The matrix M is the same as the matrix appearing in eq.(15) of Ref.[10]. There is the special case, called *on-shell*, of the background values a, f :

$$\bar{f} + ma + \frac{\lambda}{2}a^2 = 0 , \quad \chi f = (m + \lambda a)f = 0 , \quad (5)$$

which satisfies the field equation and makes \mathcal{L}_1 vanish. When the above background values (a, f) satisfy the on-shell condition above, V_0^{eff} reduces to

$$V_0^{eff}|_{\text{on-shell}} = \bar{f}f \geq 0 , \quad (6)$$

which shows the *positive semi-definiteness*. This shows the characteristic aspect of the supersymmetric configuration. The (classical) vacuum is given by: $f = \bar{f} = 0$, $a(m + \frac{\lambda}{2}a) = 0$ ($a = 0$ or $a = -\frac{2}{\lambda}m$). In the following, except when explicitly stated, we do *not* require the on-shell condition (5). We regard a and f not as specific constants (specific vacuum) but as the general source (external) fields appearing in the effective potential. It is an off-shell generalization but is the most natural one based on the background field method.

Let us now evaluate the 1-loop quantum effect. First we can integrate out the auxiliary quantum-fields F and \bar{F} using a "squaring" equation: $\bar{F}F + \chi AF + \bar{\chi}\bar{A}\bar{F} = (\bar{F} + \chi A)(F + \bar{\chi}\bar{A}) - \bar{\chi}\chi\bar{A}A$. Then the quadratic-part Lagrangian \mathcal{L}_2 reduces to \mathcal{L}'_2 :

$$\begin{aligned} \mathcal{L}'_2 &= i\partial_m \bar{\psi}_{\dot{\alpha}} (\bar{\sigma}^m)^{\dot{\alpha}\beta} \psi_{\beta} + \bar{A} \square A \\ &- \frac{1}{2} \begin{pmatrix} \psi^{\alpha} & \bar{\psi}_{\dot{\alpha}} \end{pmatrix} \begin{pmatrix} \chi \delta_{\alpha}^{\beta} & 0 \\ 0 & \bar{\chi} \delta_{\dot{\beta}}^{\dot{\alpha}} \end{pmatrix} \begin{pmatrix} \psi_{\beta} \\ \bar{\psi}^{\dot{\beta}} \end{pmatrix} - \frac{1}{2} \begin{pmatrix} \bar{A} & A \end{pmatrix} \mathbf{M} \begin{pmatrix} A \\ \bar{A} \end{pmatrix} , \\ \mathbf{M} &= \begin{pmatrix} \bar{\chi}\chi & -\lambda \bar{f} \\ -\lambda f & \bar{\chi}\chi \end{pmatrix} . \end{aligned} \quad (7)$$

The eigenvalues of \mathbf{M} are given as

$$m_{+}^2 = \bar{\chi}\chi + \lambda\sqrt{\bar{f}f} \quad , \quad m_{-}^2 = \bar{\chi}\chi - \lambda\sqrt{\bar{f}f} \quad . \quad (8)$$

The contribution to the 1-loop effective potential V_{1-loop}^{eff} , from the *bosonic* part (scalar loop), is evaluated as

$$\begin{aligned} &\int \mathcal{D}\bar{A} \mathcal{D}A \exp i \int d^4x \left\{ \bar{A} \square A - \frac{1}{2} \begin{pmatrix} \bar{A} & A \end{pmatrix} \mathbf{M} \begin{pmatrix} A \\ \bar{A} \end{pmatrix} \right\} \\ &= [\det(\square - m_{+}^2)(\square - m_{-}^2)]^{-\frac{1}{2}} = \exp \left\{ -\frac{1}{2} \text{Tr} \sum_{i=+,-} \ln(1 - \frac{m_i^2}{\square}) \right\} \\ &= \exp \left\{ -i \int d^4x V_{1-loop}^{eff} \right\} . \end{aligned} \quad (9)$$

The V_{1-loop}^{eff} above lacks the *fermionic* 1-loop contribution:

$$\begin{aligned} &\int \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp i \int d^4x \left\{ i\partial_m \bar{\psi}_{\dot{\alpha}} (\bar{\sigma}^m)^{\dot{\alpha}\beta} \psi_{\beta} - \frac{1}{2} \begin{pmatrix} \psi^{\alpha} & \bar{\psi}_{\dot{\alpha}} \end{pmatrix} \begin{pmatrix} \chi \delta_{\alpha}^{\beta} & 0 \\ 0 & \bar{\chi} \delta_{\dot{\beta}}^{\dot{\alpha}} \end{pmatrix} \begin{pmatrix} \psi_{\beta} \\ \bar{\psi}^{\dot{\beta}} \end{pmatrix} \right\} \\ &= [\det(\square - \bar{\chi}\chi)]^{+1} . \end{aligned} \quad (10)$$

This part does *not* depend on f and \bar{f} . It says the 1-loop effective potential calculated only by the scalar part is correct *up to the f -independent terms*. As far as the f -dependent part is concerned, the scalar part result (9) is sufficient. If we trace the source of this phenomenon, it is simply that the auxiliary fields f and \bar{f} have the *higher physical dimension*, M^2 . They *cannot* have the Yukawa coupling with fermions. ($F\psi\psi$ has the mass dimension 5.) This fact means

that dV_{1-loop}^{eff}/df (or $dV_{1-loop}^{eff}/d\bar{f}$) is definitely determined *only by the scalar part*. Miller[10, 17] utilized this fact, that is, F-tadpole or D-tadpole [11] in general SUSY theories are rather simply obtained. In the present case, (1-loop) F-tadpole corresponds to $dV_{1-loop}^{eff}/d\bar{f}$. He noticed, if the SUSY is preserved in the quantization, the f -independent part can be fixed by the following *boundary condition*.⁴ We follow Miller's idea. Looking at the tree-level (on-shell) result (6), and taking into account the quantum stableness of the SUSY theory, we are allowed to take the *supersymmetric boundary condition*:

$$\text{The SUSY effective potential vanishes at } f = 0. \quad (11)$$

Normalizing at $f = 0$, the 1-loop effective potential is finally obtained as

$$\begin{aligned} V_{1-loop}^{eff} - V_{1-loop}^{eff}|_{f=0} &= \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \ln \left(1 - \frac{\lambda^2 \bar{f}f}{(k^2 + \bar{\chi}\chi)^2} \right) \\ &\approx -\frac{1}{2} \lambda^2 \bar{f}f \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 + m^2)^2} + O(\lambda^3) \quad . \end{aligned} \quad (12)$$

In this simple example, we can explicitly see the subtracting term $-V_{1-loop}^{eff}|_{f=0}$ is just given by the fermion-loop contribution (10). The SUSY condition recovers the ignored (1-loop) contribution in V^{eff} . The middle expression of (12) is the same as eq.(26) of Ref.[10] The *quadratic* divergences appearing in the intermediate stages, as in (9) and (10), cancel and the *logarithmic* divergence only remain in the final expression (12). It is absorbed by the *wave-function renormalization* of the auxiliary fields as follows.⁵ In order to do the renormalization, we first introduce a *counterterm* $\Delta\mathcal{L}$ in the following form.

$$\begin{aligned} V_R^{1-loop} &\equiv V_{1-loop}^{eff} - V_{1-loop}^{eff}|_{f=0} - \Delta\mathcal{L} \quad , \\ V_R &\equiv V_0^{eff} + V_R^{1-loop} \quad , \quad V_0^{eff} = -\bar{f}f + \dots \\ \Delta\mathcal{L} &= \Delta Z \bar{f}f \quad , \end{aligned} \quad (13)$$

where $Z \equiv 1 + \Delta Z$ is the wave function renormalization factor of f and \bar{f} . The 0th (classical) part, V_0^{eff} , is added. Now we fix ΔZ by *demanding* the following *renormalization condition*.⁶

$$-1 \equiv \frac{dV_R}{d(\bar{f}f)} \Big|_{f=\bar{f}=0, a=\bar{a}=0}$$

⁴This reminds us of the similar situation of 2D WZNW model and 2D induced gravity. Polyakov and Wiegman[18] obtained the former "effective action" not by integrating the quantum field fluctuation but by solving the *chiral anomaly* equation in 2D QED. Polyakov[19] obtained the latter "effective action" by solving the *Weyl anomaly* equation. They treated the 'gauge-field tadpole' ($\delta\Gamma/\delta A_\mu$) and the 'Weyl-mode tadpole' ($\delta\Gamma/\delta\sigma = g^{\mu\nu}\delta\Gamma/\delta g^{\mu\nu}$) respectively.

⁵No divergences for the coupling operator, λAAF , and the mass term, mAF , are consistent with the *non-renormalization theorem*(see a textbook [20]). The F-term part does not receive radiative correction. See, for example, the West's textbook[20].

⁶We follow ref.[9] in the choice of the renormalization condition. In (14), by setting the coefficient in front of the term $\bar{f}f$, appearing in the effective (renormalized) potential, we define the present renormalization. No new mass parameter (such as the renormalization point) is introduced.

$$= -1 - \frac{1}{2}\lambda^2 \int_{|k| \leq \Lambda} \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + m^2)^2} + \Delta Z \quad ,$$

$$\text{hence} \quad Z \equiv 1 + \Delta Z = 1 - \frac{\lambda^2}{16\pi^2} \ln \frac{\Lambda}{m} \quad , \quad (14)$$

where Λ is the *momentum cut-off* $|k^2| \leq \Lambda^2$. The *anomalous dimension* of the auxiliary field is given by

$$F_b = \sqrt{Z}F \quad ,$$

$$\text{anomalous dimension of } F : \quad \gamma_F = \frac{\partial}{\partial \ln \Lambda} \ln Z = -\frac{\lambda^2}{16\pi^2} + O(\lambda^4) \quad . \quad (15)$$

We see the quantum effect in the SUSY theory apparently appears in the scaling behaviour of the *auxiliary field*. It implies the structure(shape) of the *effective potential* is very sensitive to the quantization. The final form, after the renormalization, is given by

$$V_R|_{\text{on-shell}} = (V_0^{eff} + V_R^{1-loop})|_{\text{on-shell}} \quad ,$$

$$V_0^{eff}|_{\text{on-shell}} = \bar{f}f \quad ,$$

$$V_R^{1-loop}|_{\text{on-shell}} = \frac{1}{64\pi^2} \left[-\lambda^2 \bar{f}f - (\bar{\chi}\chi)^2 \ln \left\{ \frac{(\bar{\chi}\chi)^2}{(\bar{\chi}\chi - \lambda\sqrt{\bar{f}f})(\bar{\chi}\chi + \lambda\sqrt{\bar{f}f})} \right\} \right. \\ \left. + 2\lambda \bar{\chi}\chi \sqrt{\bar{f}f} \ln \left\{ \frac{\bar{\chi}\chi + \lambda\sqrt{\bar{f}f}}{\bar{\chi}\chi - \lambda\sqrt{\bar{f}f}} \right\} + \lambda^2 \bar{f}f \ln \left\{ \frac{(\bar{\chi}\chi - \lambda\sqrt{\bar{f}f})(\bar{\chi}\chi + \lambda\sqrt{\bar{f}f})}{m^4} \right\} \right] \quad ,$$

$$\text{where} \quad \bar{f}f = |ma + \frac{\lambda}{2}a^2|^2 \quad , \quad \bar{\chi}\chi = |m + \lambda a|^2 \quad . \quad (16)$$

For the pure imaginary case of $a = ib$ (b is a real number), the above potentials, $V_0^{eff}|_{\text{on-shell}}$ and $V_R^{1-loop}|_{\text{on-shell}}$, are depicted in Fig.1. In this case we have $\bar{f}f = m^2 b^2 + \frac{\lambda^2}{4} b^4$, $\bar{\chi}\chi = m^2 + \lambda^2 b^2$. The precise shape of the quantum correction depends on the renormalization condition. However some characteristic features are considered meaningful. The shape of the 1-loop correction is *not* the Coleman-Weinberg type. The total shape of $V_R^{1-loop}|_{\text{on-shell}}$ is similar to the tree potential $V_0^{eff}|_{\text{on-shell}}$. This shows that the SUSY invariant vacuum, $b = 0$, is *stable* against the quantum effect.⁷ The *positive definiteness* is preserved after the 1-loop correction. The form of the potential does not essentially change. This result typically shows a general feature of SUSY theories. It was confirmed before in the counter-term calculation[21].

Super QED is similarly treated in Appendix A. In this case the matter sector is vector-like by introducing a pair of chiral multiplets. The *SUSY boundary condition* is taken at $D=0$. The anomalous dimension of the D-field and the

⁷Here we should be careful for the meaning of $V_R|_{\text{on-shell}}$. It is still an off-shell quantity in the sense that the true vacuum is realized at $b = 0$ only. Only at this vacuum, SUSY is preserved. We call $V_R|_{\text{on-shell}}$ the SUSY-invariant effective action because it is, at its vacuum, SUSY-invariant.

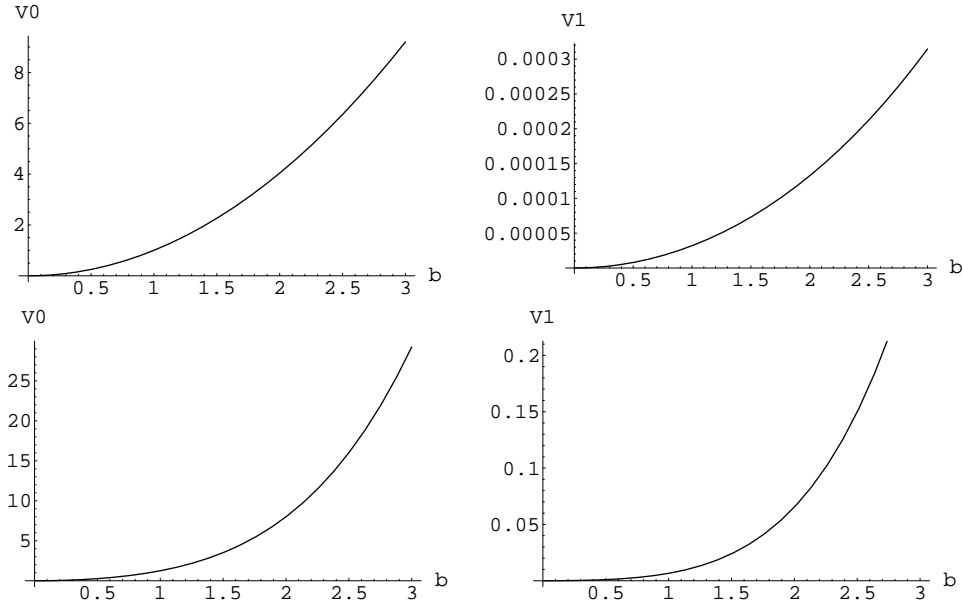


Figure 1: The effective potential of the Wess-Zumino model(16). The case $a = ib(b: \text{real})$ and the mass parameter $m = 1$. The tree part ($V_0^{eff}|_{\text{on-shell}} \equiv V_0$, left) and the 1-loop correction part ($V_R^{1-loop}|_{\text{on-shell}} \equiv V_1$, right) are depicted for the coupling parameter $\lambda = 0.1$ (up), $\lambda = 1$ (down). The horizontal axis is b . The potentials are even functions of b .

β -function of the coupling are obtained. The 1-loop effective potential is explicitly obtained and its SUSY invariant properties are confirmed. The result will become an important reference in the analysis of the bulk-boundary theory in the following sections.

3 Mirabelli-Peskin Model

As a toy model of a bulk-boundary model, Mirabelli and Peskin proposed the following system. Let us consider the 5 dimensional space-time. The space of the fifth component is taken to be S_1 , with the *periodicity* $2l$.

$$x^5 \rightarrow x^5 + 2l \quad . \quad (17)$$

We also require the system to be (anti)symmetric with respect to the Z_2 -symmetry:

$$Z_2 \text{ transformation : } x^5 \rightarrow -x^5 \quad . \quad (18)$$

This makes the two points, $x^5 = 0$ and $x^5 = l$, fixed points under Z_2 -transformation. The extra space is S^1/Z_2 orbifold. Let us consider 5D bulk theory \mathcal{L}_{bulk} which is coupled with 4D matter theory \mathcal{L}_{bnd} on a "wall" at $x^5 = 0$ and with \mathcal{L}'_{bnd} on the other "wall" at $x^5 = l$. The boundary Lagrangians are, in the bulk action, described by the delta-functions along the extra axis x^5 .

$$S = \int d^5x \{ \mathcal{L}_{bulk} + \delta(x^5) \mathcal{L}_{bnd} + \delta(x^5 - l) \mathcal{L}'_{bnd} \} \quad . \quad (19)$$

We make use of the SUSY symmetry in order to make the problem simple. Both bulk and boundary quantum effects are taken into account.

(i) 5D super Yang-Mills theory

We take, as the bulk dynamics, the 5D super YM theory which is made of a vector field A^M ($M = 0, 1, 2, 3, 5$), a scalar field Φ , a doublet of symplectic Majorana fields λ^i ($i = 1, 2$), and a triplet of auxiliary scalar fields X^a ($a = 1, 2, 3$). The metric is $(\eta_{MN}) = \text{diag}(-1, 1, 1, 1, 1)$. We basically follow the notation of [22].

$$\begin{aligned} \mathcal{L}_{SYM} &= \text{tr} \left(-\frac{1}{2} F_{MN}^2 - (\nabla_M \Phi)^2 - i \bar{\lambda}_i \gamma^M \nabla_M \lambda^i + (X^a)^2 + g \bar{\lambda}_i [\Phi, \lambda^i] \right) \quad , \\ F_{MN} &= \partial_M A_N - \partial_N A_M + ig [A_M, A_N] \quad , \quad \nabla_M \Phi = \partial_M \Phi + ig [A_M, \Phi] \quad , \\ \nabla_M \lambda^i &= \partial_M \lambda^i + ig [A_M, \lambda^i] \quad , \end{aligned} \quad (20)$$

where all bulk fields are the *adjoint* representation of the gauge group G . The $SU(2)_R$ index i is raised and lowered by the anti-symmetric tensors ϵ^{ij} and ϵ_{ij} .

$$A_M = A_{M\alpha} T^\alpha \quad , \quad \Phi = \Phi_\alpha T^\alpha \quad , \quad \lambda^i = \lambda_\alpha^i T^\alpha \quad , \quad X^a = X_\alpha^a T^\alpha \quad ,$$

⁸The present notation: $[A_M, \Phi] = if^{\alpha\beta\gamma} A_{M\alpha} \Phi_\beta T^\gamma = i(A_M \times \Phi)_\gamma T^\gamma = iA_M \times \Phi$, $\text{tr} \{ [A_M, \Phi] \partial_5 \Phi \} = (i/2) f^{\alpha\beta\gamma} A_{M\alpha} \Phi_\beta \partial_5 \Phi_\gamma = i \text{tr} \{ (A_M \times \Phi) \partial_5 \Phi \}$. Hence $\nabla_M \Phi = \partial_M \Phi - g A_M \times \Phi$.

$$[T^\alpha, T^\beta] = if^{\alpha\beta\gamma} T^\gamma \quad , \quad \text{tr}(T^\alpha T^\beta) = \frac{1}{2} \delta^{\alpha\beta} \quad , \quad (21)$$

where T^α is the generator of the group and $f^{\alpha\beta\gamma}$ is the structure constant. (As for the group indices α, β, \dots , there is no distinction between the upper one and the lower one.) The bulk Lagrangian \mathcal{L}_{SYM} of (20) is invariant under the following SUSY transformation.

$$\begin{aligned} \delta_\xi A^M &= i\bar{\xi}_i \gamma^M \lambda^i \quad , \\ \delta_\xi \Phi &= i\bar{\xi}_i \lambda^i \quad , \\ \delta_\xi \lambda^i &= (\Sigma^{MN} F_{MN} + \gamma^M \nabla_M \Phi) \xi^i + i(X^a \sigma^a)^i_j \xi^j \quad , \\ \delta_\xi X^a &= \bar{\xi}_i (\sigma^a)^i_j \gamma^M \nabla_M \lambda^j + i[\Phi, \bar{\xi}_i (\sigma^a)^i_j \lambda^j] \quad , \end{aligned} \quad (22)$$

where $\Sigma^{MN} = \frac{1}{4}[\gamma^M, \gamma^N]$, and the SUSY global parameter ξ^i is the symplectic Majorana spinor. This system has the symmetry of 8 real super charges.⁹

As the 5D gauge-fixing term, we take the Feynman gauge:

$$\mathcal{L}_{gauge} = -\text{tr}(\partial_M A^M)^2 = -\frac{1}{2}(\partial_M A^M_\alpha)^2 \quad . \quad (23)$$

The corresponding ghost Lagrangian is given by

$$\mathcal{L}_{ghost} = -2 \text{tr} \partial_M \bar{c} \cdot \nabla^M (A) c = -2 \text{tr} \partial_M \bar{c} \cdot (\partial^M c + ig[A^M, c]) \quad , \quad (24)$$

where c and \bar{c} are the complex ghost fields. We take the following bulk action.

$$\mathcal{L}_{blk} = \mathcal{L}_{SYM} + \mathcal{L}_{gauge} + \mathcal{L}_{ghost} \quad . \quad (25)$$

(ii) Z_2 -structure

In order to consistently project out $\mathcal{N} = 1$ SUSY multiplet which has 4 real super charges(4Q's), we make use of the Z_2 symmetry (18) which divides the 8 Q 's system into two 4Q's systems. We *assign* Z_2 -parity, even ($P = +1$) or odd ($P = -1$) under the Z_2 -transformation, to all fields in accordance with the 5D SUSY (22). A *consistent choice* is given as in Table 1. Note that A^m , ($m = 0, 1, 2, 3$), is the 4D components of the bulk vector A^M . A symplectic Majorana field is expressed by two Weyl spinors. We can write λ^i and ξ^i as follows:

$$\lambda^1 = \begin{pmatrix} (\lambda_L)_\alpha \\ (\bar{\lambda}_R)^{\dot{\alpha}} \end{pmatrix} \quad , \quad \lambda^2 = \begin{pmatrix} (\lambda_R)_\alpha \\ -(\bar{\lambda}_L)^{\dot{\alpha}} \end{pmatrix} \quad , \quad \xi^1 = \begin{pmatrix} (\xi_L)_\alpha \\ (\bar{\xi}_R)^{\dot{\alpha}} \end{pmatrix} \quad , \quad \xi^2 = \begin{pmatrix} (\xi_R)_\alpha \\ -(\bar{\xi}_L)^{\dot{\alpha}} \end{pmatrix} \quad . \quad (26)$$

⁹Two Dirac spinors ξ^1 and ξ^2 has a "reality" condition. The total number of the independent real SUSY-freedom is 8, which is the same as that of one Dirac spinor

	$P = +1, \xi_L$	$P = -1, \xi_R$
A^M	A^m	A^5
Φ	—	Φ
λ^i	λ_L	λ_R
X^a	X^3	$X^{1,2}$

Table 1 Z_2 – parity assignment.

\mathcal{L}_{SYM} of (20) is invariant under the Z_2 -transformation (18). On the wall ($x^5 = 0$), all *odd-parity* states vanish. The parity odd fields, A^5 and Φ in Table 1, will play an important role in the effective potential. The SUSY transformation (22) reduces to the following one of *even-parity* states generated by ξ_L :

$$\begin{aligned}
\delta_\xi A^m &= i\bar{\xi}_L \bar{\sigma}^m \lambda_L + i\xi_L \sigma^m \bar{\lambda}_L \quad , \\
\delta_\xi \lambda_L &= \sigma^{mn} F_{mn} \xi_L + i(X^3 - \nabla_5 \Phi) \xi_L \quad , \\
\delta_\xi (X^3 - \nabla_5 \Phi) &= \bar{\xi}_L \bar{\sigma}^m \nabla_m \lambda_L - \xi_L \sigma^m \nabla_m \bar{\lambda}_L \quad .
\end{aligned} \tag{27}$$

(Note that the odd parity field Φ appears in the x^5 -derivative form.) This is a $\mathcal{N} = 1$ (4D) vector multiplet ($A^m, \lambda_L, X^3 - \nabla_5 \Phi$) transformation in the *WZ-gauge* although all fields depend on the extra coordinate x^5 . Especially $\mathcal{D} \equiv X^3 - \nabla_5 \Phi$ plays the role of D-field. The multiplet defined in the 5D world can be expressed in the form of the 4D vector superfield.

$$V = -\theta \sigma^m \bar{\theta} A_m + i\theta^2 \bar{\theta} \bar{\lambda}_L - i\bar{\theta}^2 \theta \lambda_L + \frac{1}{2} \theta^2 \bar{\theta}^2 \mathcal{D} \quad , \quad \mathcal{D} = X^3 - \nabla_5 \Phi \quad . \tag{28}$$

The odd-parity fields ($\Phi + iA_5, -i\sqrt{2}\lambda_R, X^1 + iX^2$) transform as a chiral (adjoint) multiplet.

$$\begin{aligned}
\delta_\xi (\Phi + iA_5) &= \sqrt{2} \xi_L (-i\sqrt{2} \lambda_R) \quad , \\
\delta_\xi \lambda_R &= (i\sigma^m F_{5m} - \sigma^m \nabla_m \Phi) \bar{\xi}_L + i(X^1 + iX^2) \xi_L \quad , \\
\delta_\xi (X^1 + iX^2) &= 2\bar{\xi}_L (\bar{\sigma}^m \nabla_m \lambda_R - i\nabla_5 \bar{\lambda}_L) - 2i[\Phi, \bar{\xi}_L \bar{\lambda}_L] \quad .
\end{aligned} \tag{29}$$

This multiplet can be expressed in the 4D superspace as

$$\Sigma = (\Phi + iA_5) + \sqrt{2}\theta(-i\sqrt{2}\lambda_R) + \theta^2(X^1 + iX^2) \quad . \tag{30}$$

We will not use this multiplet in this paper.

iii) Matter Lagrangian on the wall

Let us introduce matter fields on the walls. We consider two cases: a) chiral matter, b) vector-like matter.

iiia) Chiral matter

We introduce, on the $x^5 = 0$ brane, a 4 dim chiral multiplet (ϕ, ψ, F) where ϕ is

a complex scalar field, ψ is a Weyl spinor and F is an auxiliary field of complex scalar. This is the simplest case as a matter candidate and was taken in the original theory[6]. The chiral superfield Θ is introduced :

$$\Theta = \phi + \sqrt{2}\theta\psi + \theta^2 F \quad , \quad \bar{\Theta} = \phi^\dagger + \sqrt{2}\bar{\theta}\bar{\psi} + \bar{\theta}^2 F^\dagger \quad . \quad (31)$$

Using the $\mathcal{N} = 1$ SUSY property of the bulk fields ($A^m, \lambda_L, \mathcal{D} = X^3 - \nabla_5 \Phi$), we can find the bulk-boundary coupling.

$$\begin{aligned} \mathcal{L}_{bnd}^{(a)} = \bar{\Theta} e^{2gV} \Theta \Big|_{\theta^2 \bar{\theta}^2} = & -\nabla_m \phi^\dagger \nabla^m \phi - i\bar{\psi} \bar{\sigma}^m \nabla_m \psi + F^\dagger F \\ & + \sqrt{2}ig(\bar{\psi} \bar{\lambda}_L \phi - \phi^\dagger \lambda_L \psi) + g\phi^\dagger \mathcal{D} \phi \quad , \end{aligned} \quad (32)$$

where $\nabla_m \equiv \partial_m + igA_m$, $\mathcal{D} = X^3 - \nabla_5 \Phi$. All matter fields are taken to be the *fundamental* representation of the internal group G. We may add the following superpotential term to the above Lagrangian.

$$\mathcal{L}_{SupPot} = \left(\frac{1}{2} m_{\alpha'\beta'} \Theta_{\alpha'} \Theta_{\beta'} + \frac{1}{3!} \lambda_{\alpha'\beta'\gamma'} \Theta_{\alpha'} \Theta_{\beta'} \Theta_{\gamma'} \right) \Big|_{\theta^2} + \text{h.c.} \quad , \quad (33)$$

where the primed Greek suffixes (α', β', \dots) show those of the fundamental representation.

iiib) Vector-like matter

We introduce, as the 4D matter fields on the $x^5 = 0$ brane, one pair of 4 dim chiral multiplets, $\Theta_S = (\phi_S, \psi_S, F_S)$ and $\Theta_R = (\phi_R, \psi_R, F_R)$.

$$\begin{aligned} \mathcal{L}_{bnd}^{(b)} = & (\bar{\Theta}_S e^{2gV} \Theta_S + \bar{\Theta}_R e^{-2gV} \Theta_R) \Big|_{\theta^2 \bar{\theta}^2} + m(\Theta_S \Theta_R \Big|_{\theta^2} + \bar{\Theta}_S \bar{\Theta}_R \Big|_{\bar{\theta}^2}) \\ = & -(\nabla_m^+ \phi_S)^\dagger (\nabla_m^+ \phi_S) - (\nabla_m^- \phi_R)^\dagger (\nabla_m^- \phi_R) + F_S^\dagger F_S + F_R^\dagger F_R \\ & - \frac{1}{2} (\bar{\psi}_S i \bar{\sigma}^m \nabla_m^+ \psi_S + \bar{\psi}_R i \bar{\sigma}^m \nabla_m^- \psi_R + \text{h.c.}) \\ & + \sqrt{2}g(i\bar{\psi}_S \bar{\lambda}_L \phi_S - i\bar{\psi}_R \bar{\lambda}_L \phi_R + \text{h.c.}) \\ & + g(\phi_S^\dagger D \phi_S - \phi_R^\dagger D \phi_R) + m\{\phi_S F_R + \phi_R F_S - \psi_S \psi_R + \text{h.c.}\} \quad , \end{aligned} \quad (34)$$

where $\nabla_m^\pm = \partial_m \pm igA_m$. This is the bulk-boundary generalization of super QED or QCD. We can identify the matter fermions (ψ_S, ψ_R) as one Dirac fermion ("electron, quark").

On the other brane $x^5 = l$, we introduce another WZ-multiplet(s), (ϕ', ψ', F') for case a), and (ϕ'_S, ψ'_S, F'_S) and (ϕ'_R, ψ'_R, F'_R) for case b). The bulk-boundary couplings are fixed in the same way. The quadratic (kinetic) terms of the vector A^m , the gaugino spinor λ_L and $\mathcal{D} = X^3 - \nabla_5 \Phi$ are in the *bulk* world. We note here the interaction between the bulk fields and the boundary ones is *definitely fixed* from SUSY. In the ordinary standpoint of the field theory, the boundary theory (32) or (34) is *perturbatively unrenormalizable* because the coupling g has the physical dimension of $M^{-1/2}$ (unrenormalizable coupling).

4 Quantization Using Background Field Method

From the results of Sect.2 (and App.A), we may put, for the purpose of obtaining the 1-loop effective potential, the following conditions on \mathcal{L}_{blk} :

$$A^m = 0 \quad (m = 0, 1, 2, 3) \quad , \quad \lambda^i = \bar{\lambda}^i = 0 \quad . \quad (35)$$

Here the extra (fifth) component of the bulk vector A^5 does *not* taken to be zero because it is regarded as a $4D$ scalar on the wall. Then $\mathcal{L}_{blk} = \mathcal{L}^{SYM} + \mathcal{L}_{gauge} + \mathcal{L}_{ghost}$ reduces to

$$\begin{aligned} & \mathcal{L}_{blk}^{red}[\Phi, X^3, A_5] \\ = & \text{tr} \left\{ -\partial_M \Phi \partial^M \Phi + X^3 X^3 - \partial_M A_5 \partial^M A_5 + 2g(\partial_5 \Phi \times A_5) \Phi - g^2(A_5 \times \Phi)(A_5 \times \Phi) \right\} \\ & + \mathcal{L}_{ghost}^{red}[c, \bar{c}, A_5] + \text{irrel. terms} \quad , \\ & \mathcal{L}_{ghost}^{red}[c, \bar{c}, A_5] = -2\text{tr} \left\{ \partial_m \bar{c} \cdot \partial^m c + \partial_5 \bar{c} \cdot (\partial^5 c + ig[A^5, c]) \right\} \quad , \quad (36) \end{aligned}$$

where we have dropped terms of $X_\alpha^1 X_\alpha^1, X_\alpha^2 X_\alpha^2$ as 'irrelevant terms' because they decouple from other fields. As for the boundary part, we may impose the conditions:

$$\begin{aligned} \text{a. Chiral matter : } & A^m = 0 \quad , \quad \psi = 0 \quad , \quad \lambda_L = 0 \quad ; \\ \text{b. vector-like matter : } & A^m = 0 \quad , \quad \psi_S = \psi_R = 0 \quad , \quad \lambda_L = 0 \quad . \quad (37) \end{aligned}$$

\mathcal{L}_{bnd} reduces to

$$\begin{aligned} & \text{a. Chiral matter} \\ & \mathcal{L}_{bnd(a)}^{red}[\phi, \phi^\dagger, \mathcal{D} = X^3 - \nabla_5 \Phi] = \\ & -\partial_m \phi^\dagger \partial^m \phi + g(X_\alpha^3 - \partial_5 \Phi_\alpha + g f^{\alpha\beta\gamma} A_{5\beta} \Phi_\gamma) \phi_{\beta'}^\dagger (T^\alpha)_{\beta'\gamma'} \phi_{\gamma'} + F^\dagger F \\ & + \left\{ \frac{m_{\alpha'\beta'}}{2} (\phi_{\alpha'} F_{\beta'} + F_{\alpha'} \phi_{\beta'}) + \frac{\lambda_{\alpha'\beta'\gamma'}}{3!} (\phi_{\alpha'} \phi_{\beta'} F_{\gamma'} + \phi_{\alpha'} F_{\beta'} \phi_{\gamma'} + F_{\alpha'} \phi_{\beta'} \phi_{\gamma'}) + \text{h.c.} \right\} \quad . \\ & \text{b. Vector-like matter} \\ & \mathcal{L}_{bnd(b)}^{red}[\phi_S, \phi_S^\dagger, \phi_R, \phi_R^\dagger, \mathcal{D} = X^3 - \nabla_5 \Phi] = -\partial_m \phi_S^\dagger \partial^m \phi_S - \partial_m \phi_R^\dagger \partial^m \phi_R \\ & + g(X_\alpha^3 - \nabla_5 \Phi_\alpha) (T^\alpha)_{\beta'\gamma'} (\phi_{S\beta'}^\dagger \phi_{S\gamma'} - \phi_{R\beta'}^\dagger \phi_{R\gamma'}) + F_S^\dagger F_S + F_R^\dagger F_R + m(\phi_S F_R + \phi_R F_S + \text{h.c.}) \quad , \quad (38) \end{aligned}$$

where α', β' are the suffixes of the fundamental representation. In the same way, the boundary Lagrangians at $x^5 = l$, $\mathcal{L}'_{bnd(a)}$ and $\mathcal{L}'_{bnd(b)}$, reduce to $\mathcal{L}'_{bnd(a),(b)}^{red} = (\phi \rightarrow \phi', F \rightarrow F' \text{ in (38)})$. Now we expand all scalar fields $(\Phi, X^3, A_5; \phi, F, \phi', F')$, except ghost fields, into the *quantum fields* (which are denoted again by the same symbols) and the *background fields* $(\varphi, \chi^3, a_5; \eta, f, \eta', f')$.

$$\begin{aligned} & \Phi \rightarrow \varphi + \Phi \quad , \quad X^3 \rightarrow \chi^3 + X^3 \quad , \quad A_5 \rightarrow a_5 + A_5 \quad , \\ & \left\{ \begin{array}{ll} \phi \rightarrow \eta + \phi, F \rightarrow f + F \text{ and primed ones} & \text{chiral matter} \\ \phi_S \rightarrow \eta_S + \phi_S, \phi_R \rightarrow \eta_R + \phi_R, F_S \rightarrow f_S + F_S, F_R \rightarrow f_R + F_R & \text{vector-like matter} \end{array} \right. \quad (39) \end{aligned}$$

We treat the ghost fields as quantum ones.

In Sec.2, for the purpose of obtaining the effective potential we consider the case that the background fields are constant (in order to pick up the non-derivative part of the effective action). In the present case of the 5D space-time, we have the extra coordinate x^5 . Because 4D(x^m -space) scalar property is independent of the extra space, we take into account the x^5 -dependency of the vacuum configuration. The distribution along the extra coordinate is important to make the localization configuration. We require that *the background fields may be constant only in 4D world, not necessarily in 5D world*. We may allow the background field to depend on the extra coordinate x^5 . This gives us an interesting possibility to the extra space model. (See also the beginning paragraph of Sec.6 where the necessity of the present treatment is explained using an explicitly- x^5 -dependent solution (62).)

When the background fields $(\varphi, \chi^3, a_5; \eta, f, \eta', f')$ satisfy the field equations derived from $\mathcal{L}_{blk}^{red} + \delta(x^5)\mathcal{L}_{bnd}^{red} + \delta(x^5 - l)\mathcal{L}_{bnd}'^{red}$, using (36) and (38), the situation is called "on-shell". The equations (on-shell condition) are given as,

$$\begin{aligned}
& \delta\Phi_\alpha \quad ; \\
& \partial_5^2 \varphi_\alpha + g f_{\beta\gamma\alpha} \partial_5 \varphi_\beta a_{5\gamma} - g f_{\alpha\beta\gamma} \partial_5 (a_{5\beta} \varphi_\gamma) - g^2 f_{\beta\alpha\tau} f_{\gamma\delta\tau} a_{5\beta} a_{5\gamma} \varphi_\delta \\
& + g \partial_5 \delta(x^5) \eta^\dagger T^\alpha \eta + g \partial_5 \delta(x^5 - l) \eta'^\dagger T^\alpha \eta' + g^2 (\delta(x^5) \eta^\dagger T^\gamma \eta + \delta(x^5 - l) \eta'^\dagger T^\gamma \eta') f^{\alpha\beta\gamma} a_{5\beta} \\
& = -\partial_5 Z_\alpha - g(Z \times a_5)_\alpha = 0, \\
& \delta A_{5\alpha} \quad ; \\
& \partial_5^2 a_{5\alpha} + g f_{\beta\alpha\gamma} \partial_5 \varphi_\beta \varphi_\gamma - g^2 f_{\alpha\beta\tau} f_{\gamma\delta\tau} \varphi_\beta a_{5\gamma} \varphi_\delta + g^2 (\delta(x^5) \eta^\dagger T^\gamma \eta + \delta(x^5 - l) \eta'^\dagger T^\gamma \eta') f^{\alpha\beta\gamma} \varphi_\beta \\
& = \partial_5^2 a_{5\alpha} - g(\varphi \times Z)_\alpha = 0, \\
& \delta X_\alpha^3 \quad ; \\
& \chi_\alpha^3 + g(\delta(x^5) \eta^\dagger T^\alpha \eta + \delta(x^5 - l) \eta'^\dagger T^\alpha \eta') = 0, \\
& \delta\phi_{\alpha'}^\dagger \quad (\delta\phi_{\alpha'}'^\dagger) \quad ; \\
& d_\beta (T^\beta \eta)_{\alpha'} + m_{\alpha'\beta'} f_{\beta'}^\dagger + \frac{1}{2} \lambda_{\alpha'\beta'\gamma'} \eta_{\beta'}^\dagger f_{\gamma'}^\dagger = 0, \quad (\eta \rightarrow \eta', f \rightarrow f' \text{ in the left eq.}) \\
& \delta F_{\alpha'}^\dagger \quad (\delta F_{\alpha'}'^\dagger) \quad ; \\
& f_{\alpha'} + m_{\alpha'\beta'} \eta_{\beta'}^\dagger + \frac{1}{2} \lambda_{\alpha'\beta'\gamma'} \eta_{\beta'}^\dagger \eta_{\gamma'}^\dagger = 0, \quad (\eta \rightarrow \eta', f \rightarrow f' \text{ in the left eq.}) \quad , (40)
\end{aligned}$$

where $d_\alpha = (\chi^3 - \partial_5 \varphi + g a_5 \times \varphi)_\alpha$ is the background (4 dimensional) D-field and $Z_\alpha \equiv -g(\delta(x^5) \eta^\dagger T^\alpha \eta + \delta(x^5 - l) \eta'^\dagger T^\alpha \eta') - \partial_5 \varphi_\alpha + g f_{\alpha\beta\gamma} a_{5\beta} \varphi_\gamma$. In deriving the above equations, we assume, based on the statement of the previous paragraph on the background field, $\varphi = \varphi(x^5), \chi^3 = \chi^3(x^5), a_5 = a_5(x^5), \eta = \text{const}, \eta' = \text{const}$. The total symmetricity of $m_{\alpha'\beta'}$ and $\lambda_{\alpha'\beta'\gamma'}$ with respect to the suffixes is also assumed. In the above derivation, we use the fact that total divergences

vanish from the *periodicity condition*. As stated below (6), we do *not* assume the above on-shell condition except when we state its use. We regard the background fields as general external fields or as off-shell fields.

The *quadratic* part w.r.t. the quantum fields $(\Phi, X^3, A_5; \phi, F, \phi', F')$ gives us the 1-loop quantum effect. That part of \mathcal{L}_{blk}^{red} is given as

$$\begin{aligned}\mathcal{L}_{blk}^2[\Phi, A_5, X^3] &= \text{tr} \{ -\partial_M \Phi \partial^M \Phi + X^3 X^3 - \partial_M A_5 \partial^M A_5 \} \\ &\quad + 2g \text{tr} [(\partial_5 \varphi \times A_5) \Phi + (\partial_5 \Phi \times a_5) \Phi + (\partial_5 \Phi \times A_5) \varphi] \\ &\quad - 2g^2 \text{tr} [(a_5 \times \varphi)(A_5 \times \Phi)] - g^2 \text{tr} (a_5 \times \Phi + A_5 \times \varphi)^2 + \mathcal{L}_{ghost}^2[c, \bar{c}] \quad , \\ \mathcal{L}_{ghost}^2[c, \bar{c}] &= -2 \text{tr} \{ \partial_m \bar{c} \cdot \partial^m c + \partial_5 \bar{c} \cdot (\partial^5 c + ig[a^5, c]) \} \quad . \quad (41)\end{aligned}$$

The quadratic part of \mathcal{L}_{bnd}^{red} is given by

$$\begin{aligned}\mathcal{L}_{bnd(a)}^2[\phi, \phi^\dagger, F, F^\dagger; \Phi, A_5, X^3] &= -\partial_m \phi^\dagger \partial^m \phi + g d_\alpha \phi^\dagger T^\alpha \phi - ig^2 [A_5, \Phi]_\alpha \eta^\dagger T^\alpha \eta \\ &\quad + g(X_\alpha^3 - \partial_5 \Phi_\alpha - ig[a_5, \Phi]_\alpha - ig[A_5, \varphi]_\alpha)(\eta^\dagger T^\alpha \phi + \phi^\dagger T^\alpha \eta) + F^\dagger F \\ &\quad + \{ m_{\alpha'\beta'} \phi_{\alpha'} F_{\beta'} + \frac{\lambda_{\alpha'\beta'\gamma'}}{2} (\phi_{\alpha'} \phi_{\beta'} f_{\gamma'} + 2\phi_{\alpha'} \eta_{\beta'} F_{\gamma'}) + \text{h.c.} \} \quad , \\ d_\alpha &\equiv (\chi^3 - \partial_5 \varphi - ig[a_5, \varphi])_\alpha \quad , \\ \mathcal{L}_{bnd(b)}^2 &= -\partial_m \phi_S^\dagger \partial^m \phi_S - \partial_m \phi_R^\dagger \partial^m \phi_R + g \{ d_\alpha (\phi_S^\dagger T^\alpha \phi_S - \phi_R^\dagger T^\alpha \phi_R) - ig[A_5, \Phi]_\alpha (\eta_S^\dagger T^\alpha \eta_S - \eta_R^\dagger T^\alpha \eta_R) \\ &\quad + (X_\alpha^3 - \partial_5 \Phi_\alpha - ig[a_5, \Phi] - ig[A_5, \varphi]) (\eta_S^\dagger T^\alpha \phi_S + \phi_S^\dagger T^\alpha \eta_S - \eta_R^\dagger T^\alpha \phi_R - \phi_R^\dagger T^\alpha \eta_R) \} \\ &\quad + F_S^\dagger F_S + F_R^\dagger F_R + m(\phi_S F_R + \phi_R F_S + \text{h.c.}) \quad , \quad (42)\end{aligned}$$

where $\phi^\dagger T^\gamma \phi \equiv \phi_{\alpha'}^\dagger (T^\gamma)_{\alpha'\beta'} \phi_{\beta'}$. $\mathcal{L}_{bnd(a)}'^2$ and $\mathcal{L}_{bnd(b)}'^2$ are the same as $\mathcal{L}_{bnd(a)}^2$ and $\mathcal{L}_{bnd(b)}^2$ except the replacement: $\phi \rightarrow \phi', F \rightarrow F', \eta \rightarrow \eta', f \rightarrow f'$. Now we can integrate out the auxiliary field X_α^3 in $\mathcal{L}_{blk}^2 + \delta(x^5) \mathcal{L}_{bnd}^2 + \delta(x^5 - l) \mathcal{L}_{bnd}'^2$. Using a "squaring" equation¹⁰:

$$\begin{aligned}&\frac{1}{2} X_\alpha^3 X_\alpha^3 + g \{ \delta(x^5) (\eta^\dagger T^\alpha \phi + \phi^\dagger T^\alpha \eta) + \delta(x^5 - l) (\eta'^\dagger T^\alpha \phi' + \phi'^\dagger T^\alpha \eta') \} X_\alpha^3 \\ &= \frac{1}{2} \{ X_\alpha^3 + g \delta(x^5) (\eta^\dagger T^\alpha \phi + \phi^\dagger T^\alpha \eta) + g \delta(x^5 - l) (\eta'^\dagger T^\alpha \phi' + \phi'^\dagger T^\alpha \eta') \}^2 \\ &- \frac{1}{2} g^2 \delta(x^5) \delta(0) (\eta^\dagger T^\alpha \phi + \phi^\dagger T^\alpha \eta)^2 - \frac{1}{2} g^2 \delta(x^5 - l) \delta(0) (\eta'^\dagger T^\alpha \phi' + \phi'^\dagger T^\alpha \eta')^2 \quad , \quad (43)\end{aligned}$$

we obtain the final 1-loop Lagrangian, necessary for the present purpose, as

$$\begin{aligned}S_a^2[\Phi, A_5; \phi, F] &= \int d^5 X [\mathcal{L}_{blk}^2|_{X^3=0} \\ &\quad + \delta(x^5) \{ -\partial_m \phi^\dagger \partial^m \phi + g d_\alpha (\phi^\dagger T^\alpha \phi) - g \partial_5 \Phi_\alpha (\eta^\dagger T^\alpha \phi + \phi^\dagger T^\alpha \eta) \\ &\quad + \left(m_{\alpha'\beta'} \phi_{\alpha'} F_{\beta'} + \frac{\lambda_{\alpha'\beta'\gamma'}}{2} (\phi_{\alpha'} \phi_{\beta'} f_{\gamma'} + 2\phi_{\alpha'} \eta_{\beta'} F_{\gamma'}) + \text{h.c.} \right) + F^\dagger F - \frac{g^2}{2} \delta(0) (\eta^\dagger T^\alpha \phi + \phi^\dagger T^\alpha \eta)^2 \} \\ &\quad + \delta(x^5 - l) \{ \phi \rightarrow \phi', \eta \rightarrow \eta', F \rightarrow F' \} \quad ,\end{aligned}$$

¹⁰Note the relation: $\delta(x^5) \delta(x^5 - l) = 0$.

$$\mathcal{L}_{blk}^2|_{X^3=0} = \mathcal{L}_{ghost}^2[c, \bar{c}] + \text{tr} \left\{ -\partial_M \Phi \partial^M \Phi - \partial_M A_5 \partial^M A_5 + 2g ((\partial_5 \varphi \times A_5) \Phi + (\partial_5 \Phi \times a_5) \Phi + (\partial_5 \Phi \times A_5) \varphi) \right. \\ \left. - 2g^2 (a_5 \times \varphi)(A_5 \times \Phi) - g^2 (a_5 \times \Phi + A_5 \times \varphi)^2 \right\} \quad , (44)$$

for the chiral matter model.¹¹ Similarly we obtain, for the vector-like matter, as

$$S_b^2[\Phi, A_5; \phi_S, \phi_R] = \int d^5 X \left[\mathcal{L}_{blk}^2|_{X^3=0} + \delta(x^5) \left\{ -(\partial_m \phi_S^\dagger \partial^m \phi_S + \partial_m \phi_R^\dagger \partial^m \phi_R) \right. \right. \\ \left. + g d_\alpha (\phi_S^\dagger T^\alpha \phi_S - \phi_R^\dagger T^\alpha \phi_R) - g \partial_5 \Phi_\alpha (\eta_S^\dagger T^\alpha \phi_S + \phi_S^\dagger T^\alpha \eta_S - \eta_R^\dagger T^\alpha \phi_R - \phi_R^\dagger T^\alpha \eta_R) \right. \\ \left. + F_S^\dagger F_S + F_R^\dagger F_R + m(\phi_S F_R + \phi_R F_S + \text{h.c.}) \right. \\ \left. - \frac{g^2}{2} \delta(0) (\eta_S^\dagger T^\alpha \phi_S + \phi_S^\dagger T^\alpha \eta_S - \eta_R^\dagger T^\alpha \phi_R - \phi_R^\dagger T^\alpha \eta_R)^2 \right\} + \delta(x^5 - l) \{ \phi \rightarrow \phi', \eta \rightarrow \eta', F \rightarrow F' \} \quad . (45)$$

For simplicity we consider the case of no superpotential: $m_{\alpha'\beta'} = \lambda_{\alpha'\beta'\gamma'} = 0$, hereafter.¹²

5 Bulk and Boundary Quantum Effects

Before the *full* 1-loop calculation of the next section, it is useful to look at some important diagrams appearing in the perturbation w.r.t. the coupling g . We can express propagators and vertices as in Fig.2 (for the bulk part) and Fig.3 (for the boundary and mixed parts). All double lines express the background fields. The corresponding terms in the Lagrangian, from which the Feynman rules can be easily read, are given by, for the bulk part(Fig.2),

$$(5P1) : -\frac{1}{2} \partial_M \Phi_\alpha \partial^M \Phi_\alpha \quad , \quad (5P2) : -\frac{1}{2} \partial_M A_{5\alpha} \partial^M A_{5\alpha} \quad , \\ (5V1) : g f_{\alpha\beta\gamma} \{ \partial_5 \varphi_\alpha \cdot A_{5\beta} \Phi_\gamma + \partial_5 \Phi_\alpha \cdot A_{5\beta} \varphi_\gamma \} \quad , \\ (5V2) : -g^2 f_{\alpha\beta\tau} f_{\gamma\delta\tau} a_{5\alpha} \varphi_\beta A_{5\gamma} \Phi_\delta - g^2 f_{\alpha\beta\tau} f_{\gamma\delta\tau} a_{5\alpha} \Phi_\beta A_{5\gamma} \varphi_\delta \quad , \\ (5V3) : g f_{\alpha\beta\gamma} \partial_5 \Phi_\alpha a_{5\beta} \Phi_\gamma \quad , \quad (5V4) : -\frac{g^2}{2} f_{\alpha\beta\tau} f_{\gamma\delta\tau} a_{5\alpha} \Phi_\beta a_{5\gamma} \Phi_\delta \quad , \\ (5V5) : -\frac{g^2}{2} f_{\alpha\beta\tau} f_{\gamma\delta\tau} A_{5\alpha} \varphi_\beta A_{5\gamma} \varphi_\delta \quad . \quad (46)$$

They can be read from terms in $\mathcal{L}_{blk}^2|_{X^3=0}$ of (44). Those for the boundary at $x^5 = 0$ and mixed parts (Fig.3) are given by "δ-function parts" of (44).

$$(4P) : -\delta(x^5) \partial_m \phi^\dagger \partial^m \phi \quad , \\ (4V1) : \delta(x^5) g d_\alpha \phi^\dagger T^\alpha \phi \quad , \quad (4V2) : -\frac{1}{2} g^2 \delta(x^5) \delta(0) (\eta^\dagger T^\alpha \phi)^2 \quad ,$$

¹¹Here we may omit the terms, $-ig^2[A_5, \Phi]_\alpha \eta^\dagger T^\alpha \eta$, $-ig^2([a_5, \Phi]_\alpha + [A_5, \varphi]_\alpha)(\eta^\dagger T^\alpha \phi + \phi^\dagger T^\alpha \eta)$ in $\mathcal{L}_{bnd(a)}^2$ of (42). From the Z_2 -odd property of a_5, A_5, φ and Φ , the above terms, with the term $\delta(x^5)$ or $\delta(x^5 - l)$ multiplied, have no contribution to the 1-loop effect. The same thing is used in (45).

¹²We examine the case with the superpotential in ref.[23].

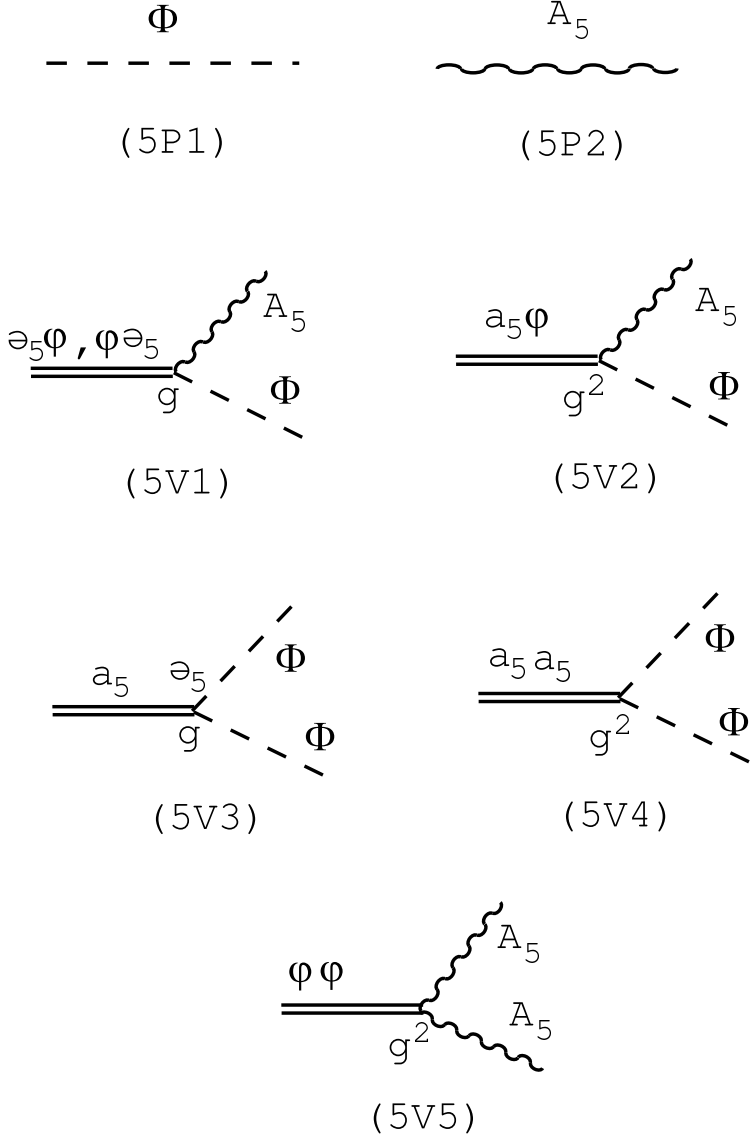


Figure 2: Feynman graphs for the bulk fields. (5P1),...(5V5) of (46).

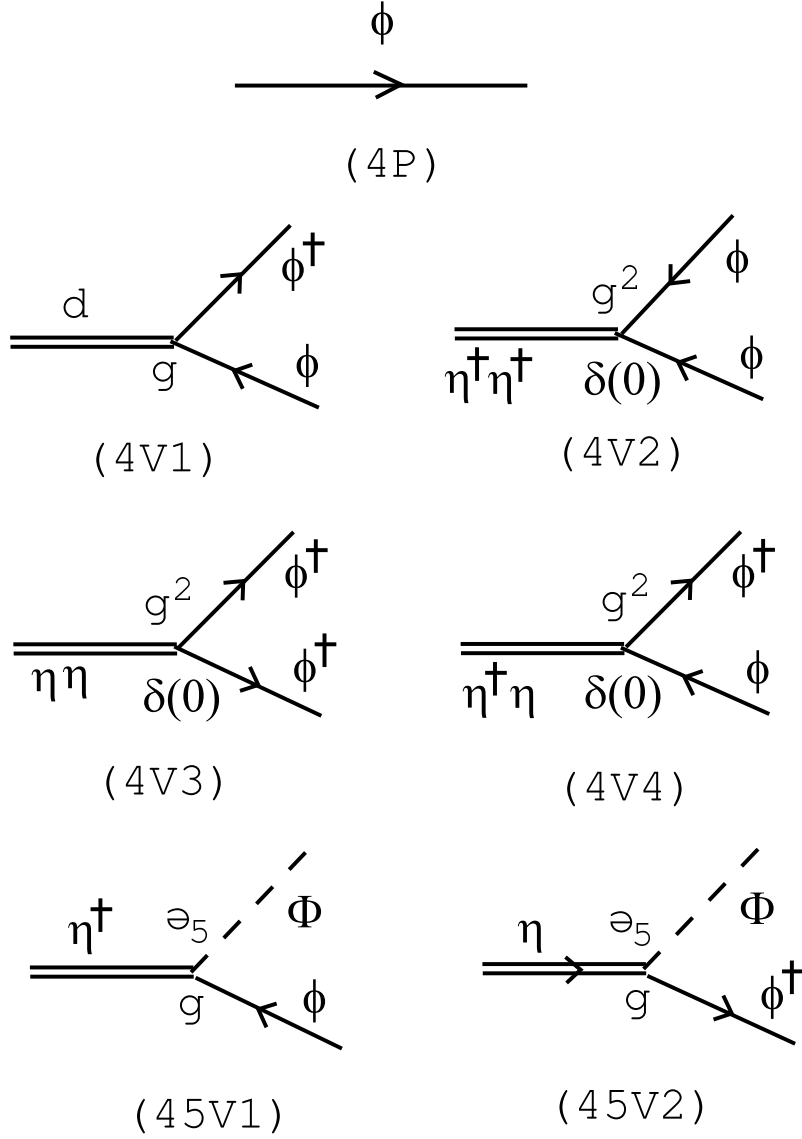


Figure 3: Feynman graphs for the boundary at $x^5 = 0$ and bulk-boundary-mixed fields. (4P),...(45V2) of (47). For all vertex graphs, the overall factor, $\delta(x^5)$, is omitted. See (47) for detail.

$$\begin{aligned}
(4V3) &: -\frac{1}{2}g^2\delta(x^5)\delta(0)(\phi^\dagger T^\alpha \eta)^2 \quad , \\
(4V4) &: -g^2\delta(x^5)\delta(0)(\eta^\dagger T^\alpha \phi)(\phi^\dagger T^\alpha \eta) \quad , \\
(45V1) &: -\delta(x^5)g\partial_5\Phi_\alpha\eta^\dagger T^\alpha \phi \quad , \quad (45V2) : -\delta(x^5)g\partial_5\Phi_\alpha\phi^\dagger T^\alpha \eta \quad . \quad (47)
\end{aligned}$$

Those for the boundary at $x^5 = l$ and mixed parts are the same as above except the replacement: $\delta(x^5) \rightarrow \delta(x^5 - l)$, $\phi \rightarrow \phi'$, $\eta \rightarrow \eta'$.

(i) Boundary (4D) Quantum Effect

All divergent diagrams (for the chiral matter model) up to the order of g^3 are listed up in Fig.4. All are 1-loop diagrams within the $x^5 = 0$ brane.

The diagram (a) is interesting because its presence says Fayet-Iliopoulos D-term appears in the boundary due to the *radiative correction*. It is *quadratically* divergent. The term is proportional to $\text{Tr} T^\alpha$, hence it exists *only when the gauge group G involves $U(1)$* . If the appearance really happens it could give a dynamical SUSY breaking (see a textbook[24]). (Note that the tadpole diagram of massless field in 4D vanishes in the *dimensional regularization* [14]¹³. Hence the presence of the FI D-term is rather subtle.) This D-term does *not* appear in the vector-like matter case, $\mathcal{L}_{bnd}^{(b)}$, because the ϕ_S and ϕ_R contributions cancel each other. (The situation is the same as the super QED.)

The diagram (b) was considered in ref.[6]. It contributes, with (f) of Fig.5 explained later, to the self-energy of the scalar matter. This diagram (b) is independent of d , hence does *not* contribute to the effective potential under the SUSY boundary condition.

The diagram (c) gives the renormalization of D-field. The tree part is in the bulk as $\text{tr}(X^3 X^3 - \partial_5 \Phi \partial^5 \Phi)$. (The corresponding part appears in Super QED. See d^2 -term of eq.(105).)

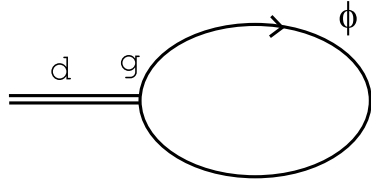
The diagram (d) gives, with (g) of Fig.5, the renormalization of the gauge coupling g , and contributes to the β -function $\beta(g)$. (This part is very contrasting with the corresponding part of Super QED (*daa*-term). We will discuss it in the final part of this section as (g)/Fig.5+(d)/Fig.4 part)

The contribution to the effective potential of each diagram (of Fig.4) is given by

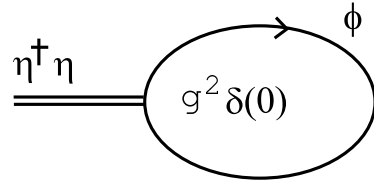
$$\begin{aligned}
\text{(a) and (b)} &: i\{gd_\alpha(T^\alpha)_{\beta'\beta'} - g^2\delta(0)(\eta^\dagger T^\alpha)_{\beta'}(T^\alpha \eta)_{\beta'}\} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} \quad , \\
\text{(c)} &: \frac{i^2}{2!}g^2 \cdot \frac{1}{2}d_\alpha d_\alpha \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2)^2} \quad , \\
\text{(d)} &: \frac{i^2}{2!} \cdot 2 \cdot (-g^3\delta(0))d_\alpha(\eta^\dagger T^\beta T^\alpha T^\beta \eta) \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2)^2} \quad . \quad (48)
\end{aligned}$$

(ii) Bulk (5D) Quantum Effect

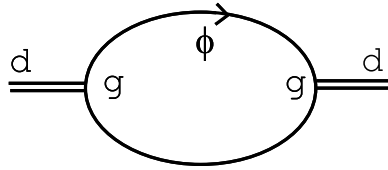
¹³On the other hand, the dimensional regularization is generally considered non-appropriate for the SUSY theories because the totally anti-symmetric tensor $\epsilon_{\mu\nu\lambda\sigma}$ is essentially involved with SUSY symmetry[25]. This looks to obscure the presence of the valid calculation of the tadpole diagram.



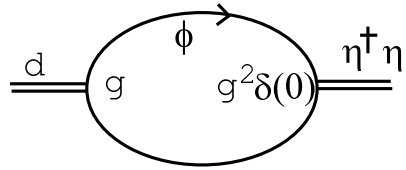
(a)



(b)



(c)



(d)

Figure 4: Divergent Feynman graphs for the boundary part up to the order of g^3 .

Among bulk quantum propagations, we do not, at present, consider those which propagate between one brane and the other brane. Those diagrams play an important role as the gauge mediation mechanism in ref.[6]. The present interest, however, is not the SUSY breaking mechanism. This simplification is admitted because we can control the ignored contribution by adjusting the length l between the two branes.

All divergent diagrams are listed up in Fig.5 up to the order of g^3 . Only the diagram (g) contributes, others do not in the SUSY boundary condition.¹⁴ The diagram (f) was analysed in Ref.[6] for the calculation of the matter-field self energy. The diagram (g) gives the bulk contribution to the β -function of the coupling g .

The contribution to the effective potential of diagrams (f) and (g) are given by

$$\begin{aligned} \text{(f)} &: g^2(\eta^\dagger T^\alpha T^\alpha \eta) \frac{i^2}{2!} \cdot 2 \int_{k^5} \frac{(k^5)^2}{k^2 + (k^5)^2} \frac{1}{k^2} \ , \\ \text{(g)} &: g^3(\eta^\dagger T^\alpha T^\gamma T^\alpha \eta) d_\gamma (-1) \int_{k^5} \frac{(k^5)^2}{-k^2 - (k^5)^2} \frac{1}{(k^2)^2} \ , \end{aligned} \quad (49)$$

where

$$\int_{k^5} \equiv \int \frac{d^4 k}{(2\pi)^4} \frac{1}{2l} \sum_{k^5 \in \frac{\pi}{2}\mathbf{Z}} \ . \quad (50)$$

The k^5 -summation comes from the KK-expansion for the bulk field Φ , which will be explained in (65). The result (f) of (49) is consistent with the corresponding term in (24) and (25) of ref.[6].

Using a formula

$$\sum_{n \in \mathbf{Z}} \frac{1}{x^2 + n^2} = \frac{\pi}{x} \coth(\pi x) \ , \quad (51)$$

the k^5 summation part in (f) and (g) can be evaluated as ($k^0 = ik^4, k^2 = (k^1)^2 + (k^2)^2 + (k^3)^2 + (k^4)^2 \equiv \bar{k}^2$)

$$\begin{aligned} \sum_{k^5 \in \frac{\pi}{2}\mathbf{Z}} \frac{(k^5)^2}{-k^2 - (k^5)^2} &= - \sum_{k^5 \in \frac{\pi}{2}\mathbf{Z}} \frac{(k^5)^2}{\bar{k}^2 + (k^5)^2} \\ &= - \left(\sum_{n \in \mathbf{Z}} 1 \right) + \bar{k}^2 \left(\frac{l}{\pi} \right)^2 \sum_{n \in \mathbf{Z}} \frac{1}{\bar{k}^2 \left(\frac{l}{\pi} \right)^2 + n^2} \\ &= -2l\delta(0) + l\sqrt{\bar{k}^2} \coth(l\sqrt{\bar{k}^2}) \ , \end{aligned} \quad (52)$$

¹⁴ $\mathcal{D} = X^3 - \nabla_5 \Phi$ is defined in the bulk. It plays the role of D-field on the boundary theory \mathcal{L}_{bnd} . Its background field $d = \chi^3 - \nabla_5(a)\varphi$ can be taken independt of $-\nabla_5(a)\varphi$. The purely bulk diagrams (e) and (h) (of Fig.5), which contains $\partial_5 \varphi$, are treated as d-independent ones. They do not contribute the effective potential in the SUSY boundary condition.

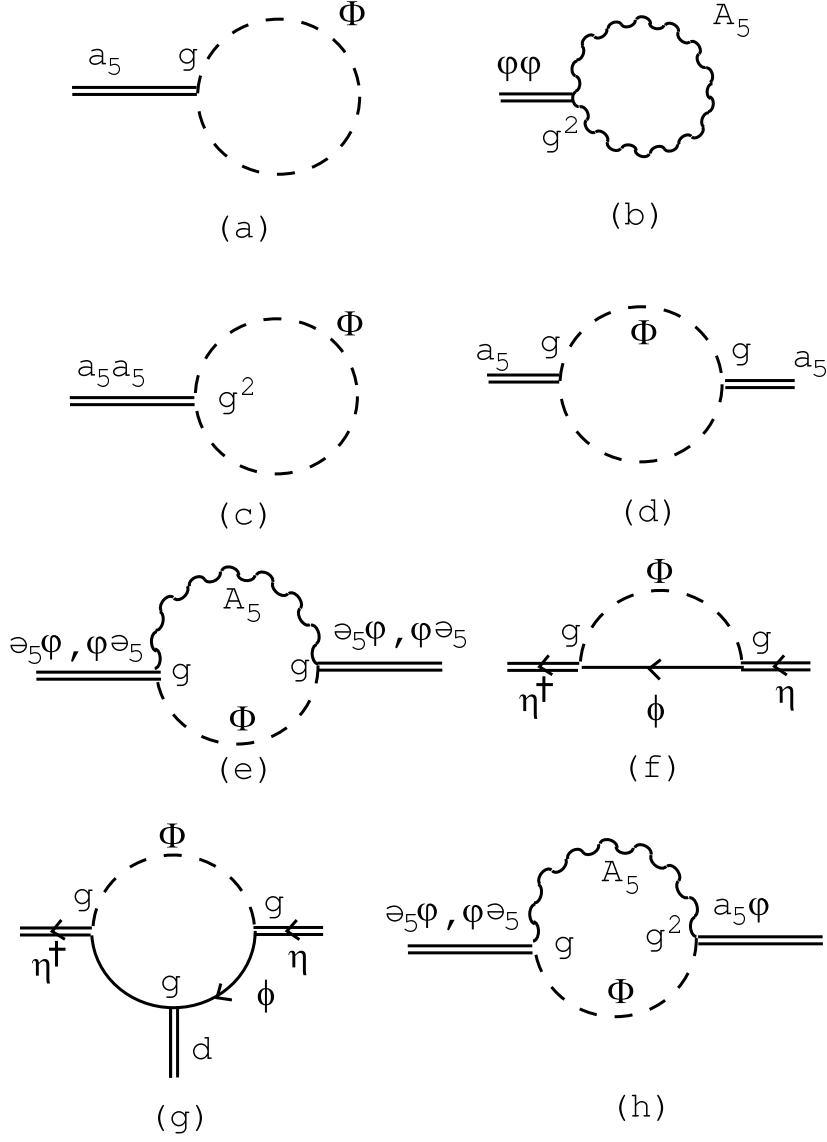


Figure 5: Divergent Feynman graphs for the bulk part up to the order of g^3 .

where the *Wick-rotation* of k^0 -axis is done and the relation

$$\sum_{n \in \mathbf{Z}} 1 = 2l\delta(0) \quad , \quad (53)$$

is used. The above result tells us the contribution from the *singular* parts in the boundary, that is, $\delta(0)$ parts in (b) and (d) of eq.(48) (see also Fig.4), *cancel* those in the bulk, that is, (f) and (g) of (49) (see also Fig.5). This phenomenon was pointed out for the self-energy diagrams in [6]. In App.B, it is shown that the cancellation phenomenon more generally (at the full order of the coupling within 1-loop) occurs in the effective potential.¹⁵ The final results are obtained as

$$\begin{aligned} \text{(f)/Fig.5+(b)/Fig.4} &: -g^2(\eta^\dagger T^\alpha T^\alpha \eta) \int \frac{id^4 \bar{k}}{(2\pi)^4} \frac{1}{2} \sqrt{\bar{k}^2} \coth(l\sqrt{\bar{k}^2}) \frac{1}{-\bar{k}^2} \quad , \\ \text{(g)/Fig.5+(d)/Fig.4} &: -g^3(\eta^\dagger T^\alpha T^\gamma T^\alpha \eta) d_\gamma \int \frac{id^4 \bar{k}}{(2\pi)^4} \frac{1}{2} \sqrt{\bar{k}^2} \coth(l\sqrt{\bar{k}^2}) \frac{1}{(-\bar{k}^2)^2} \quad . \end{aligned} \quad (54)$$

We note the above results give correct 4D expressions in the limit of $l\sqrt{\bar{k}^2} \ll 1$: $l\sqrt{\bar{k}^2} \coth(l\sqrt{\bar{k}^2}) \rightarrow 1$. (See the *da* \bar{a} -part of Super QED, (105).) Hence the present bulk-boundary system can be regarded as some "deformation" of the corresponding 4D theory.¹⁶

If we write the main "deformation" factor as follows

$$\coth(l\sqrt{\bar{k}^2}) = 1 - 2 \sum_{n=0}^{\infty} e^{-2nl\sqrt{\bar{k}^2}} \quad , \quad (55)$$

the *role of the extra space* becomes clear. Because the spectrum above (" E_n " = $n\sqrt{\bar{k}^2}$) shows that of the harmonic oscillator in the temperature $T = 1/(2l)$, it can be translated as "the whole (4D Euclidean) system is exposed to the heat-bath and in the equilibrium state with temperature $T = 1/(2l)$ ".¹⁷ The size of the extra space gives the *inverse temperature*. The $l\sqrt{\bar{k}^2} \ll 1$ limit in the previous paragraph corresponds to the *high temperature limit*.

Here the role of the *singular* term becomes clear. It is a "*counterterm*" to cancel the divergences coming from the *KK-mode summation*. The (f)+(b) part is independent of d , hence it does *not* contribute to the SUSY effective potential.¹⁸ (We expect (f)+(b) part is cancelled by the vector- and spinor-loop contribution.) This fact implies the above "smoothing" phenomenon takes place *independently of the SUSY requirement*. We should note that, after the above cancellation, divergences, due to 4D-momentum integral, still remain. They correspond to the ordinary divergences due to the (SUSY) local interaction.

¹⁵In ref.[2], $\delta(x^5)$ is called "classical singularity" and the treatment of its divergence $\delta(0)$ is discussed using a "generalized" renormalization group.

¹⁶The similar propagator form appears also in 5D bulk (AdS₅) approach at some limit[4].

¹⁷The same thing is commented in ref.[4] from the AdS₅ approach.

¹⁸This is consistent with $\mathcal{N} = 1$ SUSY non-renormalization theorem.

Let us obtain renormalization group quantities from the previous result, $\eta^\dagger T^\gamma \eta d_\gamma$ -term. The "deformed" propagator still makes the ultraviolet behaviour *worse* than the usual 4D propagator.

$$l \times \int \frac{d^4 \bar{k}}{(2\pi)^4} \sqrt{\bar{k}^2} \coth(l\sqrt{\bar{k}^2}) \frac{1}{(\bar{k}^2)^2} = \frac{1}{16\pi^2} \{l\Lambda - \ln 2 - \ln \sinh l\epsilon\} \quad , \quad (56)$$

where Λ and ϵ is the ultra-violet and infra-red cut-offs respectively, $\epsilon \leq |\bar{k}| \leq \Lambda$. (We do not care about the infra-red divergence because it can be cured by taking the massive matter multiplet.) It is *linearly* divergent. This result is reasonable from the power counting. Now we consider the scaling behaviour of the gauge coupling in the renormalization procedure. This problem is generally hard because the 5D gauge theory is regarded (perturbatively) *nonrenormalizable*. In the present case, however, we expect this model reduces to a 4D SUSY gauge theory in the limit $l \rightarrow 0$. We precisely define the limit as

$$\begin{aligned} \frac{g^2}{l} &\equiv \alpha \quad (\text{fixed}) \ll 1 \quad , \\ l \rightarrow 0 \quad , \quad g \rightarrow 0 \quad , \quad g^2 &\ll l \quad , \end{aligned} \quad (57)$$

where the *dimensionless* coupling α is introduced instead of g^2 . From the relation $g^2 \Lambda = \alpha \cdot l \Lambda$, we are naturally led to introduce *another cut-off* $\tilde{\Lambda}$ instead of Λ as

$$l\Lambda \equiv \ln \tilde{\Lambda} \quad . \quad (58)$$

This relation connects two transformations, *scaling* and *translation*: $\tilde{\Lambda} \rightarrow \tilde{\Lambda} e^\nu$ (scaling) versus $\Lambda \rightarrow \Lambda + \nu/l$ (translation). Then the renormalization group β -function of the dimensionless coupling α is obtained as

$$\begin{aligned} g_b &= g + \Delta g = g \left(1 + \frac{1}{8 \times 16\pi^2} \alpha \ln \tilde{\Lambda}\right) \quad , \\ \alpha_b &= \frac{1}{l} (g + \Delta g)^2 = \alpha \left(1 + \frac{1}{4 \times 16\pi^2} \alpha \ln \tilde{\Lambda} + O(\alpha^2)\right) \quad , \\ 0 &= \frac{d}{d(\ln \tilde{\Lambda})} \ln \alpha_b = (1 + O(\alpha)) \frac{d}{d(\ln \tilde{\Lambda})} \ln \alpha + \frac{1}{4 \times 16\pi^2} \alpha + O(\alpha^2) \quad , \\ \beta_\alpha &= \frac{d}{d(\ln \tilde{\Lambda})} \ln \alpha = -\frac{1}{4 \times 16\pi^2} \alpha \quad , \end{aligned} \quad (59)$$

where g_b and α_b are bare quantities and $G=\text{SU}(2)$ is taken ($T^\alpha T^\gamma T^\alpha = -T^\gamma/4$, see eq.(113)). The above result coincides with that of the ordinary 4D chiral-gauge SUSY theory (See textbooks[20, 26]). We confirm here that the correct 4D renormalization works although 5D quantum loop expression (56) is linearly divergent.

The previous paragraph confirms that the renormalization procedure works well at the 4D limit. In this paragraph, we argue a "renormalization" procedure for the general case of l (not limited to the $l \rightarrow 0$ case) and propose a practical

approach to define a finite quantity from a divergent one (such as (54), (56)) coming from the 5D quantum effect. In the present standpoint the extra axis is regarded as a *regularization* axis.¹⁹ We have already pointed out that the *macro* parameter $1/l$ plays a role of the temperature $T = 1/(2l)$ to smooth the UV behaviour. We recall a historically-famous fact in the beginning of the quantum mechanics. In the Planck distribution of the energy spectrum in a cavity, the light behaves like a wave in the high temperature region compared with its own energy (Rayleigh-Jeans's region, $k_B T \geq h\nu$, where k_B and h are the Boltzman constant and the Planck constant respectively) whereas it behaves like a particle in the low temperature region (Wien's region, $k_B T \leq h\nu$). In the present case, we are treating 5D quantum dynamics which can be regarded as a system composed of 4D Euclidean system of "light" (oscillator) with energy $\sqrt{\vec{k}^2}$. They are thermally distributed in the heat-bath with the temperature $T = 1/(2l)$. Now we take a natural restriction on the present treatment of the 5D quantum field theory: We cannot treat it in the Wien's region because, in this region, "the light" behaves like a particle and some new mass (energy) unit (probably the Planck mass) should exist in the theory. At present, however, we do not have such mass unit and it implies the present *field theory* treatment breaks down in the Wien's region. (In other words, "quantization" in the "phase" space of Λ and l is lacking.) We must switch to an unknown treatment in order to obtain a meaningful quantity from the divergent ones (54), (56). Let us propose a condition on the 4D momentum cut-off Λ . We should choose Λ in such a way that *the structure of the extra space* (the circle in the present case) *can not be recognized in the 4D world*.

$$\Lambda \lesssim T = \frac{1}{2l} \quad . \quad (60)$$

If we adopt this idea²⁰, the integral (56) becomes well-defined ($l\Lambda \lesssim 1$). Here we propose a sort of "renormalization" for the present 5D quantum system. Note here that the *UV cut-off* Λ , of the *4D momentum* \vec{k} , is essentially given by the inverse of the *IR cut-off* parameter l of the *extra space*. In this way, we have the following relation

$$|\vec{k}| \leq \Lambda \lesssim \frac{1}{2l} = T \quad . \quad (61)$$

This is a sort of "mass hierarchy" relation which appears in extra-dimensional models. The relation (61) reminds us of the similar one that appears in the regularization of fermion determinant (the domain wall fermion or the overlap formalism) in the lattice field theory. (See (32) of ref.[30], eq.(29) of ref.[31],

¹⁹The standpoint is the same as that appeared in the domain wall fermion of the lattice field theory.[27, 28, 29]

²⁰A comparative treatment was proposed by Randall and Schwarz [3, 4]. They examined the UV-divergence problem in 5D Yang-Mills theory on the AdS_5 geometry. In the analysis, 4D-momentum/extra-space-coordinate propagator is taken. They take such a regularization that the 4D-momentum cut-off depends on the extra coordinate. They claim the linear divergence reduces to log-divergence.

and eq.(26) of ref.[32].) The chiral symmetry in the fermion system corresponds to Z^2 -symmetry in the present case.

In this section, we have confirmed that the renormalization works well as far as the 4D world is concerned. Aside from the (5D) renormalization problem, we next examine the vacuum structure.

6 Vacuum in the Brane World and Mass Matrix

First we examine the vacuum in the present 5D approach. The relevant scalars are a_5 and φ , that is, the background fields of A_5 (the extra component of the bulk vector) and Φ (the bulk scalar) respectively. They should be, in principle, given by solving the (renormalized) equation of motion. They describe the vacuum. We usually take the following procedure in order to obtain a vacuum.

[*Ordinary* procedure for the vacuum search[33]]

- 1) First we obtain the effective potential assuming the *scalar* property of the vacuum (as described in (35,37)) and the *constancy* of the scalar vacuum expectation values.
- 2) Take the minimum of the effective potential.

At the present case, however, we should take into account the x^5 -*dependency* and the Z_2 -*property* of the vacuum expectation value. We take the following forms of $a_5(x^5)$ and $\varphi(x^5)$, which describe the *localized* (around $x^5 = 0, l$) configuration and a natural generalization²¹ of the ordinary treatment stated above.

$$a_{5\gamma}(x^5) = \bar{a}_\gamma \epsilon(x^5) \quad , \quad \varphi_\gamma(x^5) = \bar{\varphi}_\gamma \epsilon(x^5) \quad ,$$

$$\epsilon(x^5) = \begin{cases} +1 & \text{for } 2nl < x^5 < (1+2n)l \\ 0 & \text{for } x^5 = nl \\ -1 & \text{for } (2n-1)l < x^5 < 2nl \end{cases} \quad n \in \mathbf{Z} \quad , \quad (62)$$

where $\epsilon(x^5)$ is the *periodic sign function* with the periodicity $2l$. \bar{a}_γ and $\bar{\varphi}_\gamma$ are positive constants. See Fig.6 and Fig.7. It is shown, in App.C, that the above forms of $a_5(x^5)$ and $\varphi(x^5)$ *satisfy the field equation* of the present model. The periodic sign function can be regarded as the *thin-wall limit* of a *kink* solution and shows the *localization* of the bulk scalar and the extra component of the bulk vector. This generalization is also natural from the viewpoint that the present theory starts with the *singular* interaction (δ -function term of (19).). We may

²¹The condition of *constant* is generalized to *piece-wise constant*. This is required from the necessity of a non-trivial vacuum and the consistency with the Z_2 odd property. We stress here the present generalization, that is, the allowance of x^5 -dependence on the vacuum scalars (a_5 and φ), makes it possible to naturally introduce the piece-wise constant ($\epsilon(x^5)$) in the theory. It is consistent with SUSY because the configuration (62) is obtained as a solution of the present SUSY theory. See App.C. This situation should be compared with that appeared in the work by Bergshoeff, Kallosh and Van Proeyen[34]. They *replace* some constants (masses, couplings, ...) with supersymmetric singlet fields which behave as piece-wise constants. They have to newly add (D-1)-form field in order to keep SUSY.

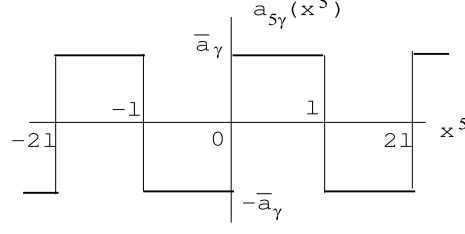


Figure 6: Behaviour of the background field $a_{5\gamma}(x^5)$ (the extra component of the bulk vector).

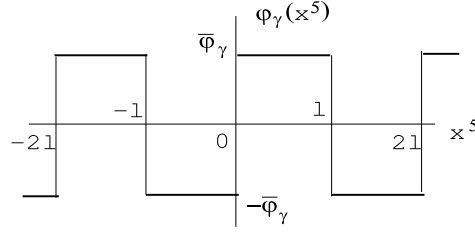


Figure 7: Behaviour of the background field $\varphi_\gamma(x^5)$ (the bulk scalar).

use the *piecewise-continuous* or *piecewise-smooth* functions as the theoretical materials, which is required from Z_2 -property[35].

We now begin to prepare for the full (with respect to the coupling order) calculation of the 1-loop effective potential. The "1-loop" action, (44), can be expressed as

$$\begin{aligned}
S_a^2 &= S_a^{free} + S^{ghost} + \int d^5 X \\
&\times \frac{1}{2} \begin{pmatrix} \phi_{\alpha'}^\dagger & \phi_{\alpha'} & \Phi_\alpha & A_{5\alpha} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} M_{\phi^\dagger \phi} & M_{\phi^\dagger \phi^\dagger} \\ M_{\phi \phi} & M_{\phi \phi^\dagger} \end{pmatrix}_{\alpha' \beta'} & \begin{pmatrix} M_{\phi^\dagger \Phi} & 0 \\ M_{\phi \Phi} & 0 \end{pmatrix}_{\alpha' \beta} \\ \begin{pmatrix} M_{\Phi \phi} & M_{\Phi \phi^\dagger} \\ 0 & 0 \end{pmatrix}_{\alpha \beta'} & \begin{pmatrix} M_{\Phi \Phi} & M_{\Phi A_5} \\ M_{A_5 \Phi} & M_{A_5 A_5} \end{pmatrix}_{\alpha \beta} \end{pmatrix} \begin{pmatrix} \phi_{\beta'} \\ \phi_{\beta'}^\dagger \\ \Phi_\beta \\ A_{5\beta} \end{pmatrix} \\
&\quad + (\phi' \text{ and } \phi'^\dagger \text{ involving terms}), \\
S_a^{free} &= \int d^5 X \left[\text{tr} \{ -\partial_M \Phi \partial^M \Phi - \partial_M A_5 \partial^M A_5 \} - \delta(x^5) \partial_m \phi^\dagger \partial^m \phi \right], \\
S^{ghost} &= - \int d^5 X \left[\partial_M \bar{c}_\alpha \cdot \partial^M c_\alpha + i g f_{\alpha\beta\gamma} \partial_5 \bar{c}_\alpha \cdot a_{5\beta} c_\gamma \right], \quad (63)
\end{aligned}$$

where each component is read from (44) as

$$\begin{aligned}
(M_{\phi^\dagger \phi})_{\alpha' \beta'} &= g \delta(x^5) d_\gamma (T^\gamma)_{\alpha' \beta'} - g^2 \delta(0) \delta(x^5) (T^\gamma \eta)_{\alpha'} (\eta^\dagger T^\gamma)_{\beta'} \quad , \\
(M_{\phi \phi^\dagger})_{\alpha' \beta'} &= g \delta(x^5) d_\gamma (T^\gamma)_{\beta' \alpha'} - g^2 \delta(0) \delta(x^5) (\eta^\dagger T^\gamma)_{\alpha'} (T^\gamma \eta)_{\beta'} \quad , \\
(M_{\phi \phi})_{\alpha' \beta'} &= -g^2 \delta(0) \delta(x^5) (\eta^\dagger T^\gamma)_{\alpha'} (\eta^\dagger T^\gamma)_{\beta'}
\end{aligned}$$

$$\begin{aligned}
(M_{\phi^\dagger \phi^\dagger})_{\alpha' \beta'} &= -g^2 \delta(0) \delta(x^5) (T^\gamma \eta)_{\alpha'} (T^\gamma \eta)_{\beta'} , \\
(M_{\Phi \phi})_{\alpha \beta'} &= (M_{\phi \Phi})_{\beta' \alpha} = g \partial_5 \delta(x^5) \cdot (\eta^\dagger T^\alpha)_{\beta'} , \\
(M_{\Phi \phi^\dagger})_{\alpha \beta'} &= (M_{\phi^\dagger \Phi})_{\beta' \alpha} = g \partial_5 \delta(x^5) \cdot (T^\alpha \eta)_{\beta'} , \\
(M_{\Phi \Phi})_{\alpha \beta} &= -g \overleftarrow{\partial}_5 f_{\alpha \beta \gamma} a_{5 \gamma} + g f_{\alpha \beta \gamma} a_{5 \gamma} \vec{\partial}_5 - g^2 f_{\alpha \delta \tau} f_{\beta \gamma \tau} a_{5 \delta} a_{5 \gamma} , \\
(M_{A_5 A_5})_{\alpha \beta} &= -g^2 f_{\alpha \gamma \tau} f_{\beta \delta \tau} \varphi_\gamma \varphi_\delta , \\
(M_{A_5 \Phi})_{\alpha \beta} &= g f_{\alpha \beta \gamma} \partial_5 \varphi_\gamma - g f_{\alpha \beta \gamma} \varphi_\gamma \vec{\partial}_5 - g^2 f_{\alpha \beta \tau} f_{\gamma \delta \tau} a_{5 \gamma} \varphi_\delta - g^2 f_{\gamma \beta \tau} f_{\alpha \delta \tau} a_{5 \gamma} \varphi_\delta , \\
(M_{\Phi A_5})_{\alpha \beta} &= -g f_{\alpha \beta \gamma} \partial_5 \varphi_\gamma + g f_{\alpha \beta \gamma} \overleftarrow{\partial}_5 \varphi_\gamma + g^2 f_{\alpha \beta \tau} f_{\gamma \delta \tau} a_{5 \gamma} \varphi_\delta - g^2 f_{\gamma \alpha \tau} f_{\beta \delta \tau} a_{5 \gamma} \varphi_\delta . \quad (64)
\end{aligned}$$

In the present analysis, as mentioned in Sect.5, we ignore the quantum propagation between the two branes. We consider only the case that the quantum-loops propagate between the $x^5 = 0$ brane and the bulk or purely within the $x^5 = 0$ brane. Hence we may ignore the $\delta(x^5 - l)$ -terms in the above expression.

From the periodicity ($x^5 \rightarrow x^5 + 2l$) and the Z_2 -odd property, the bulk fields $\Phi(X), A_5(X)$ can be KK-expanded as

$$\begin{aligned}
\Phi(x, x^5) &= \frac{1}{\sqrt{l}} \sum_{n=1}^{\infty} \Phi_n(x) \sin\left(\frac{n\pi}{l} x^5\right) , \\
A_5(x, x^5) &= \frac{1}{\sqrt{l}} \sum_{n=1}^{\infty} A_n(x) \sin\left(\frac{n\pi}{l} x^5\right) , \quad (65)
\end{aligned}$$

where the normalization is taken in the way: $\int_{-l}^l \Phi^2 dx^5 = \sum_{n=1}^{\infty} \Phi_n(x)^2$, $\int_{-l}^l A_5^2 dx^5 = \sum_{n=1}^{\infty} A_n(x)^2$.

We evaluate the action term by term.

(i) Free Part of the bulk and boundary system

The free part S_a^{free} can be obtained as

$$\begin{aligned}
S_a^{free} &= \int d^4 x \\
&\left[\sum_{k=1}^{\infty} \text{tr} \left\{ -\partial_m \Phi_k \partial^m \Phi_k - \left(\frac{k\pi}{l}\right)^2 \Phi_k^2 - \partial_m A_k \partial^m A_k - \left(\frac{k\pi}{l}\right)^2 A_k^2 \right\} - \partial_m \phi^\dagger \partial^m \phi \right] . \quad (66)
\end{aligned}$$

From the Z_2 -odd property, zero KK-mode does not appear. All quantum modes are massive with the order of l^{-1} .

(ii) $M_{\phi^\dagger \phi}, M_{\phi^\dagger \phi^\dagger}, M_{\phi \phi}, M_{\phi \phi^\dagger}$ (boundary part)

The boundary part can be read from (64).

(iii) $M_{\phi^\dagger\Phi}, M_{\phi\Phi}, M_{\Phi\phi}, M_{\Phi\phi^\dagger}$ (bulk-boundary mixed part)

$$\begin{aligned}
\frac{1}{2} \int d^5 X \phi_{\alpha'} (M_{\phi\Phi})_{\alpha'\beta} \Phi_\beta &= \frac{1}{2} \int d^5 X \Phi_\alpha (M_{\Phi\phi})_{\alpha\beta'} \phi_{\beta'} = \\
&\quad - \frac{g}{2\sqrt{l}} \int d^4 x (\eta^\dagger T^\beta \phi) \sum_{n=1}^{\infty} \frac{n\pi}{l} \Phi_{n\beta}(x) \quad , \\
\frac{1}{2} \int d^5 X \phi_{\alpha'}^\dagger (M_{\phi^\dagger\Phi})_{\alpha'\beta} \Phi_\beta &= \frac{1}{2} \int d^5 X \Phi_\alpha (M_{\Phi\phi^\dagger})_{\alpha\beta'} \phi_{\beta'}^\dagger = \\
&\quad - \frac{g}{2\sqrt{l}} \int d^4 x (\phi^\dagger T^\alpha \eta) \sum_{n=1}^{\infty} \frac{n\pi}{l} \Phi_{n\alpha}(x) \quad . \tag{67}
\end{aligned}$$

The remaining ones are bulk-bulk contribution.

(iv) $M_{\Phi\Phi}$

The one part of $\frac{1}{2} \int d^5 X \Phi_\alpha (M_{\Phi\Phi})_{\alpha\beta} \Phi_\beta \equiv S_{iv}^{int}$ is evaluated as

$$\begin{aligned}
S_{iv1}^{int} &\equiv \frac{-g^2}{2} \int d^5 X f_{\alpha\delta\tau} f_{\beta\gamma\tau} a_{5\delta} a_{5\gamma} \Phi_\alpha \Phi_\beta \\
&= -\frac{g^2}{2} f_{\alpha\delta\tau} f_{\beta\gamma\tau} \bar{a}_\delta \bar{a}_\gamma \int d^4 x \sum_{n=1}^{\infty} \Phi_{n\alpha}(x) \Phi_{n\beta}(x) \quad , \tag{68}
\end{aligned}$$

where we use $a_{5\delta} a_{5\gamma} = \bar{a}_{5\delta} \bar{a}_{5\gamma} \epsilon(x^5)^2 = \bar{a}_{5\delta} \bar{a}_{5\gamma}$. The other part can be expressed as

$$\begin{aligned}
S_{iv2}^{int} &\equiv \frac{g}{2} \int d^5 X f_{\alpha\beta\gamma} a_{5\gamma} (-\partial_5 \Phi_\alpha \Phi_\beta + \Phi_\alpha \partial_5 \Phi_\beta) \\
&= -g f_{\alpha\beta\gamma} \bar{a}_\gamma \int d^4 x \int_{-l}^l dx^5 \epsilon(x^5) \partial_5 \Phi_\alpha \cdot \Phi_\beta \quad . \tag{69}
\end{aligned}$$

Using the Fourier expansion of the periodic sign function,

$$\epsilon(x) = \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{1}{2n+1} \sin\left\{\frac{(2n+1)\pi}{l} x\right\} \quad , \tag{70}$$

we obtain

$$\begin{aligned}
S_{iv2}^{int} &= \frac{2g}{l} f_{\alpha\beta\gamma} \bar{a}_\gamma \int d^4 x \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_{m\alpha} m Q_{mn} \Phi_{n\beta} \quad , \\
&\quad \int_{-l}^l dx^5 \epsilon(x^5) \cos\left(\frac{m\pi}{l} x^5\right) \sin\left(\frac{n\pi}{l} x^5\right) = -\frac{2l}{\pi} Q_{mn} \quad ,
\end{aligned}$$

$$Q_{mn} = \begin{cases} \frac{1}{m-n} & m-n = \text{odd} \\ 0 & m-n = \text{even} \end{cases} = \begin{cases} \frac{1}{2}\{1 - (-1)^{m-n}\}\frac{1}{m-n} & m \neq n \\ 0 & m = n \end{cases} . \quad (71)$$

We note the anti-symmetry: $Q_{mn} = -Q_{nm}$.

(v) $M_{A_5 A_5}$

$$\begin{aligned} S_v^{int} &\equiv \frac{1}{2} \int d^5 X A_{5\alpha}(X) (M_{A_5 A_5})_{\alpha\beta} A_{5\beta}(X) \\ &= -\frac{g^2}{2l} f_{\alpha\gamma\tau} f_{\beta\delta\tau} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \int d^4 x A_{n\alpha}(x) A_{k\beta}(x) \times \\ &\quad \int_{-l}^l dx^5 \varphi_{\gamma}(x^5) \varphi_{\delta}(x^5) \sin\left(\frac{n\pi}{l} x^5\right) \sin\left(\frac{k\pi}{l} x^5\right) . \end{aligned} \quad (72)$$

Using the localized form of $\varphi(x^5)$ given in (62), we obtain

$$S_v^{int} = -\frac{g^2}{2} f_{\alpha\gamma\tau} f_{\beta\delta\tau} \sum_{n=1}^{\infty} \bar{\varphi}_{\gamma} \bar{\varphi}_{\delta} \int d^4 x A_{n\alpha}(x) A_{n\beta}(x) . \quad (73)$$

(vi) $M_{\Phi A_5}, M_{A_5 \Phi}$

This group consists of four terms.

$$\begin{aligned} S_{vi}^{int} &\equiv \frac{1}{2} \int d^5 X \Phi_{\alpha} (M_{\Phi A_5})_{\alpha\beta} A_{5\beta} = \frac{1}{2} \int d^5 X A_{5\alpha} (M_{A_5 \Phi})_{\alpha\beta} \Phi_{\beta} \\ &= S_{vi1}^{int} + S_{vi2}^{int} + S_{vi3}^{int} + S_{vi4}^{int} , \\ S_{vi1}^{int} &= -\frac{1}{2} \int d^5 X g f_{\alpha\beta\gamma} \Phi_{\alpha} \partial_5 \varphi_{\gamma} A_{5\beta} , \quad S_{vi2}^{int} = \frac{1}{2} \int d^5 X g f_{\alpha\beta\gamma} \partial_5 \Phi_{\alpha} \varphi_{\gamma} A_{5\beta} , \\ S_{vi3}^{int} &= \frac{1}{2} \int d^5 X g^2 f_{\alpha\beta\tau} f_{\gamma\delta\tau} \Phi_{\alpha} a_{5\gamma} \varphi_{\delta} A_{5\beta} , \quad S_{vi4}^{int} = -\frac{1}{2} \int d^5 X g^2 f_{\gamma\alpha\tau} f_{\beta\delta\tau} \Phi_{\alpha} a_{5\gamma} \varphi_{\delta} A_{5\beta} . \end{aligned} \quad (74)$$

Here we note the relation

$$\partial_5 \varphi_{\gamma} = 2\bar{\varphi}_{\gamma} \{ \delta(x^5) - \delta(x^5 - l) \} , \quad (75)$$

which expresses the *localization* of the bulk scalar. $\delta(x)$ is the periodic (periodicity $2l$) delta function. See Fig.8. Using the above equation, we can evaluate the first term as follows.

$$S_{vi1}^{int} = -g f_{\alpha\beta\gamma} \bar{\varphi}_{\gamma} \int d^4 x [\Phi_{\alpha} A_{5\beta}|_{x^5=0} - \Phi_{\alpha} A_{5\beta}|_{x^5=l}] = 0 . \quad (76)$$

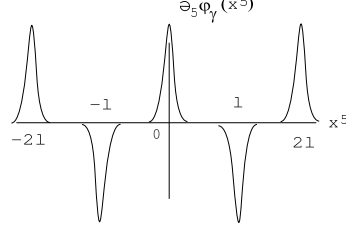


Figure 8: Behaviour of $\partial_5 \varphi_\gamma(x^5)$.

The third and fourth terms are evaluated as

$$\begin{aligned}
S_{vi3}^{int} &= \frac{g^2}{2} f_{\alpha\beta\tau} f_{\gamma\delta\tau} \bar{a}_\gamma \bar{\varphi}_\delta \sum_{n=1}^{\infty} \int d^4x \Phi_{n\alpha}(x) A_{n\beta}(x) \quad , \\
S_{vi4}^{int} &= -\frac{g^2}{2} f_{\gamma\alpha\tau} f_{\beta\delta\tau} \bar{a}_\gamma \bar{\varphi}_\delta \sum_{n=1}^{\infty} \int d^4x \Phi_{n\alpha}(x) A_{n\beta}(x) \quad .
\end{aligned} \tag{77}$$

Using the relation (71), we obtain

$$S_{vi2}^{int} = -\frac{g}{l} f_{\alpha\beta\gamma} \bar{\varphi}_\gamma \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_{m\alpha} m Q_{mn} A_{n\beta} \quad . \tag{78}$$

The background fields we take, (62), satisfy the required boundary condition. They also satisfy the on-shell condition (40) for an appropriate choice of $\bar{a}, \bar{\varphi}, \eta, \eta^\dagger$ and χ^3 . Explanation is given in App.C.

We summarize the results of (i)-(vi) as follows.

$$\begin{aligned}
& S_a^2 = S^{ghost} + \int d^4x \times \\
& \frac{1}{2} \left(\begin{array}{cccc} \phi_{\alpha'}^\dagger & \phi_{\alpha'} & \Phi_{m\alpha} & A_{m\alpha} \end{array} \right) \left(\begin{array}{cc} \left(\begin{array}{cc} \mathcal{M}_{\phi^\dagger\phi} & \mathcal{M}_{\phi^\dagger\phi^\dagger} \\ \mathcal{M}_{\phi\phi} & \mathcal{M}_{\phi\phi^\dagger} \end{array} \right)_{\alpha'\beta'} & \left(\begin{array}{cc} \mathcal{M}_{\phi^\dagger\Phi} & 0 \\ \mathcal{M}_{\phi\Phi} & 0 \end{array} \right)_{\alpha'n\beta} \\ \left(\begin{array}{cc} \mathcal{M}_{\Phi\phi} & \mathcal{M}_{\Phi\phi^\dagger} \\ 0 & 0 \end{array} \right)_{m\alpha\beta'} & \left(\begin{array}{cc} \mathcal{M}_{\Phi\Phi} & \mathcal{M}_{\Phi A} \\ \mathcal{M}_{A\Phi} & \mathcal{M}_{AA} \end{array} \right)_{m\alpha n\beta} \end{array} \right) \left(\begin{array}{c} \phi_{\beta'} \\ \phi_{\beta'}^\dagger \\ \Phi_{n\beta} \\ A_{n\beta} \end{array} \right) \\
& + (\phi' \text{ and } \phi'^\dagger \text{ involving terms}) \quad , \tag{79}
\end{aligned}$$

where the integer suffixes m and n runs from 1 to ∞ , and each component is described as

$$\begin{aligned}
\mathcal{M}_{\phi_{\alpha'}^\dagger \phi_{\beta'}} &= \partial^2 \delta_{\alpha'\beta'} + g d_\gamma (T^\gamma)_{\alpha'\beta'} - g^2 \delta(0) (T^\gamma \eta)_{\alpha'} (\eta^\dagger T^\gamma)_{\beta'} \quad , \\
\mathcal{M}_{\phi_{\alpha'}^\dagger \phi_{\beta'}^\dagger} &= -g^2 \delta(0) (T^\gamma \eta)_{\alpha'} (T^\gamma \eta)_{\beta'} \quad , \quad \mathcal{M}_{\phi_{\alpha'} \phi_{\beta'}} = -g^2 \delta(0) (\eta^\dagger T^\gamma)_{\alpha'} (\eta^\dagger T^\gamma)_{\beta'} \quad , \\
\mathcal{M}_{\phi_{\alpha'} \phi_{\beta'}^\dagger} &= \partial^2 \delta_{\alpha'\beta'} + g d_\gamma (T^\gamma)_{\beta'\alpha'} - g^2 \delta(0) (\eta^\dagger T^\gamma)_{\alpha'} (T^\gamma \eta)_{\beta'}
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{\phi_{\alpha'}^{\dagger}, \Phi_{n\beta}} &= -\frac{g}{\sqrt{l}}(T^{\beta}\eta)_{\alpha'}\frac{n\pi}{l} = \mathcal{M}_{\Phi_{n\beta}\phi_{\alpha'}^{\dagger}} \quad , \quad \mathcal{M}_{\phi_{\alpha'}\Phi_{n\beta}} = -\frac{g}{\sqrt{l}}(\eta^{\dagger}T^{\beta})_{\alpha'}\frac{n\pi}{l} = \mathcal{M}_{\Phi_{n\beta}\phi_{\alpha'}} \quad , \\
\mathcal{M}_{\Phi_{m\alpha}\Phi_{n\beta}} &= -\{-\partial^2 + (\frac{n\pi}{l})^2\}\delta_{mn}\delta_{\alpha\beta} - g^2 f_{\alpha\delta\tau} f_{\beta\gamma\tau} \bar{a}_{\delta} \bar{a}_{\gamma} \delta_{mn} + \frac{4g}{l} f_{\alpha\beta\gamma} \bar{a}_{\gamma} m Q_{mn} \quad , \\
\mathcal{M}_{\Phi_{m\alpha} A_{n\beta}} &= g^2 f_{\alpha\beta\tau} f_{\gamma\delta\tau} \bar{a}_{\gamma} \bar{\varphi}_{\delta} \delta_{mn} - g^2 f_{\gamma\alpha\tau} f_{\beta\delta\tau} \bar{a}_{\gamma} \bar{\varphi}_{\delta} \delta_{mn} - \frac{2g}{l} f_{\alpha\beta\gamma} \bar{\varphi}_{\gamma} m Q_{mn} = \mathcal{M}_{A_{n\beta}\Phi_{m\alpha}} \quad , \\
\mathcal{M}_{A_{m\alpha} A_{n\beta}} &= -\{-\partial^2 + (\frac{n\pi}{l})^2\}\delta_{mn}\delta_{\alpha\beta} - g^2 f_{\alpha\gamma\tau} f_{\beta\delta\tau} \bar{\varphi}_{\gamma} \bar{\varphi}_{\delta} \delta_{mn} \quad . \quad (80)
\end{aligned}$$

where the kinetic (free) part, S_a^{free} , is included ($\partial^2 \equiv \partial_m \partial^m$ is the 4D Laplacian) in the “Mass” matrix. The repeated indices imply the Einstein’s summation convention.²²

7 Effective Potential of Bulk-Boundary System

As shown in Sec.2 and App.A for simple models, the effective potential is obtained from the eigenvalues of the relevant mass-matrix obtained by the background expansion. Let us obtain the effective potential from the mass matrix \mathcal{M} of (79) and (80). It is composed of three field values $\eta, \eta^{\dagger}, d_{\alpha}$, two wall “heights”, $\bar{a}_{\alpha}, \bar{\varphi}_{\alpha}$, the gauge coupling, g and the boundary parameter l . The full explicit calculation, even at 1-loop level, is technically hard. We obtain some interesting “sections” of the full result: Case (A), $\eta = 0, \eta^{\dagger} = 0$; Case (B), $\bar{a} = 0, \bar{\varphi} = 0$. Detailed explanation is given in App.B.

Case (A): $\eta = 0, \eta^{\dagger} = 0$

We look the potential with the suppression of the scalar matter dependence. (Or we may say we look the potential from the $\eta = \eta^{\dagger} = 0$ point.) In this case the mass matrix \mathcal{M} has the following properties: (1) In \mathcal{M} , the boundary part and the bulk one *decouple* each other; (2) All $\delta(0)$ -terms disappear. The boundary-loop quantum effect gives rise to the following potential before the renormalization procedure:

$$V_{1-loop}^{eff} = \int \frac{d^4 k}{(2\pi)^4} \ln \left\{ 1 - \frac{g^2}{4} \frac{d^2}{(k^2)^2} \right\} \quad , \quad (81)$$

where we take $G=SU(2)$ as the internal gauge group. The behaviour is similar to the super QED explained in App.A. The above expression, when treated perturbatively, is *logarithmically* divergent. Noting the relation: $d_{\alpha} = (\chi^3 - \partial_5 \varphi + g a_5 \times \varphi)_{\alpha}$, we realize the renormalization procedure connects the boundary and the bulk phenomena through the field renormalization of X^3 and Φ although

²²For the convenience, we list the physical dimensions of various quantities. $[\eta] = [\partial_m] = M$, $[\bar{a}] = [\varphi] = M^{3/2}$, $[d] = M^{5/2}$, $[g] = M^{-1/2}$, $[l] = M^{-1}$, $[\delta(0)] = M$, $[\phi] = [\phi^{\dagger}] = [\Phi_n] = [A_n] = M$.

we do not touch on the renormalization of the bulk fields.²³

The bulk-loop quantum effect does not give the d_α -dependence to the vacuum energy. Hence it does *not* contribute to the effective potential after the use of the SUSY boundary condition. It gives, however, an important result: the scalar-loop contribution to the vacuum energy which depends on the "wall heights" \bar{a} and $\bar{\varphi}$.²⁴ (We expect some part of the contribution appear when the boson-fermion balance breaks down due to some SUSY breaking mechanism.) We can regard it as a *new type Casimir energy*, because \bar{a} and $\bar{\varphi}$ can be regarded as different-type boundary parameters from l . [36] For the large circle limit $l \rightarrow \infty$, the final result of the new Casimir energy, per one KK-mode, is

$$\frac{1}{l} V_{Casimir}^{eff} = \frac{g^2}{l^3} (\alpha_1 \bar{\varphi}^2 + \alpha_2 \bar{a}^2 + \alpha_3 \bar{a} \cdot \bar{\varphi}) + O(g^4) \quad , \quad (82)$$

where α_1, α_2 and α_3 are some constants. The new points, compared with the ordinary Casimir energy [7], are 1) the potential depends on the circle radius as l^{-3} ; 2) the potential depends on the gauge coupling g ; 3) the potential depends on the "wall heights", $\bar{\varphi}$ and \bar{a} . We expect the above quantity (82) does not depend on the gauge we have chosen [37]. This contribution from the scalar loop, however, is expected to be cancelled by those from the fermion and vector loops in the present SUSY-invariant setting.

Case (B): $\bar{a} = 0, \bar{\varphi} = 0$

In this case the brane structure disappears. The situation is similar to the case of Appelquist and Chodos (AC). From the bulk modes of Φ and A_5 , we have AC-type eigenvalues.

$$\lambda_n = -k^2 - \left(\frac{n\pi}{l}\right)^2 \quad , \quad n = 1, 2, 3, \dots \quad , \quad (83)$$

(See (136) and (146)). This gives the famous form of the Casimir energy.

$$\frac{1}{l} V_{Casimir}^{eff} = \frac{\text{const}}{l^5} \quad . \quad (84)$$

This is the scalar-loop contribution and is expected to be cancelled by other non-scalar fields effect.

The eigenvalues for the boundary part is obtained as a complicated expression involving the following terms:

$$S \equiv \eta^\dagger \eta \quad , \quad d^2 = d_\alpha d_\alpha \quad , \quad d \cdot V \equiv d_\alpha \eta^\dagger T^\alpha \eta \quad , \quad V^2 \equiv (\eta^\dagger T^\alpha \eta)^2 \quad . \quad (85)$$

We have the full expression in the computer file. In the manipulation of eigenvalues search (determinant calculation), we face the following combination of

²³The importance of the "communication" between the bulk and boundary renormalizations was stressed by Goldberger and Wise [2].

²⁴No Casimir energy in the SUSY invariant theory is reasonable from the general result about the vanishing energy of the SUSY vacuum.

terms.

$$\delta(0) + \frac{1}{l} \sum_{m=1}^{\infty} \frac{(\pi m/l)^2}{-\lambda - k^2 - (\pi m/l)^2} . \quad (86)$$

The first term comes from the singular terms in \mathcal{M} , the second from the KK-mode sum. Using the relation $\sum_{m \in \mathbf{Z}} 1 = 2l\delta(0)$, the above sum leads to a regular quantity.

$$\delta(0)|_{sm} = \frac{1}{2l} \sum_{m \in \mathbf{Z}} \frac{\lambda + k^2}{\lambda + k^2 + (\pi m/l)^2} = \begin{cases} \frac{1}{2} \sqrt{\lambda + k^2} \coth\{l\sqrt{\lambda + k^2}\} & \lambda > -k^2 \\ \frac{1}{2} \sqrt{-\lambda - k^2} \cot\{l\sqrt{-\lambda - k^2}\} & -k^2 > \lambda \end{cases} . \quad (87)$$

We have confirmed this "smoothing" phenomenon occurs at the 1-loop *full* level.

The effective potential induced on the boundary comes from the eigenvalues depending on the field d_α . When we look at d^2 -part, the following ones are obtained as the dominant part.

$$\lambda_\pm = -k^2 \pm \frac{g}{2} \sqrt{d^2} . \quad (88)$$

This is the same as Case A.

When we look at the $d \cdot V \equiv d_\alpha \eta^\dagger T^\alpha \eta$ part, the eigenvalues are dominated by the solutions of the following equation.

$$(\lambda + k^2)^2 - \frac{g^3}{2} d \cdot V \frac{\sqrt{\lambda + k^2}}{2} \coth l\sqrt{\lambda + k^2} = 0 . \quad (89)$$

In the perturbative approach, this equation gives, in the $O(g^3)$ order, two eigenvalues λ_1, λ_2 which satisfy

$$\lambda_1 \lambda_2 = (k^2)^2 \left(1 - \frac{g^3}{4} d \cdot V \frac{\sqrt{k^2} \coth l\sqrt{k^2}}{(k^2)^2} \right) , \quad (90)$$

(see (158)). This is the same as (54). The eigenvalues obtained as the full solutions of (89) gives the effective potential at the 1-loop *full* level. The corresponding diagrams are shown in Fig.9. The figure is a bulk-boundary generalization of the Coleman-Weinberg's case[9].²⁵

We succeed in obtaining the full 1-loop eigenvalues induced by the bulk-boundary quantum effect.

²⁵If we take the 4D-limit, $l\sqrt{k^2} \ll 1$, in (90), we see the result essentially reduces to the ordinary type appearing in 4D theory (such as a term, (105), in 4D Super QED).

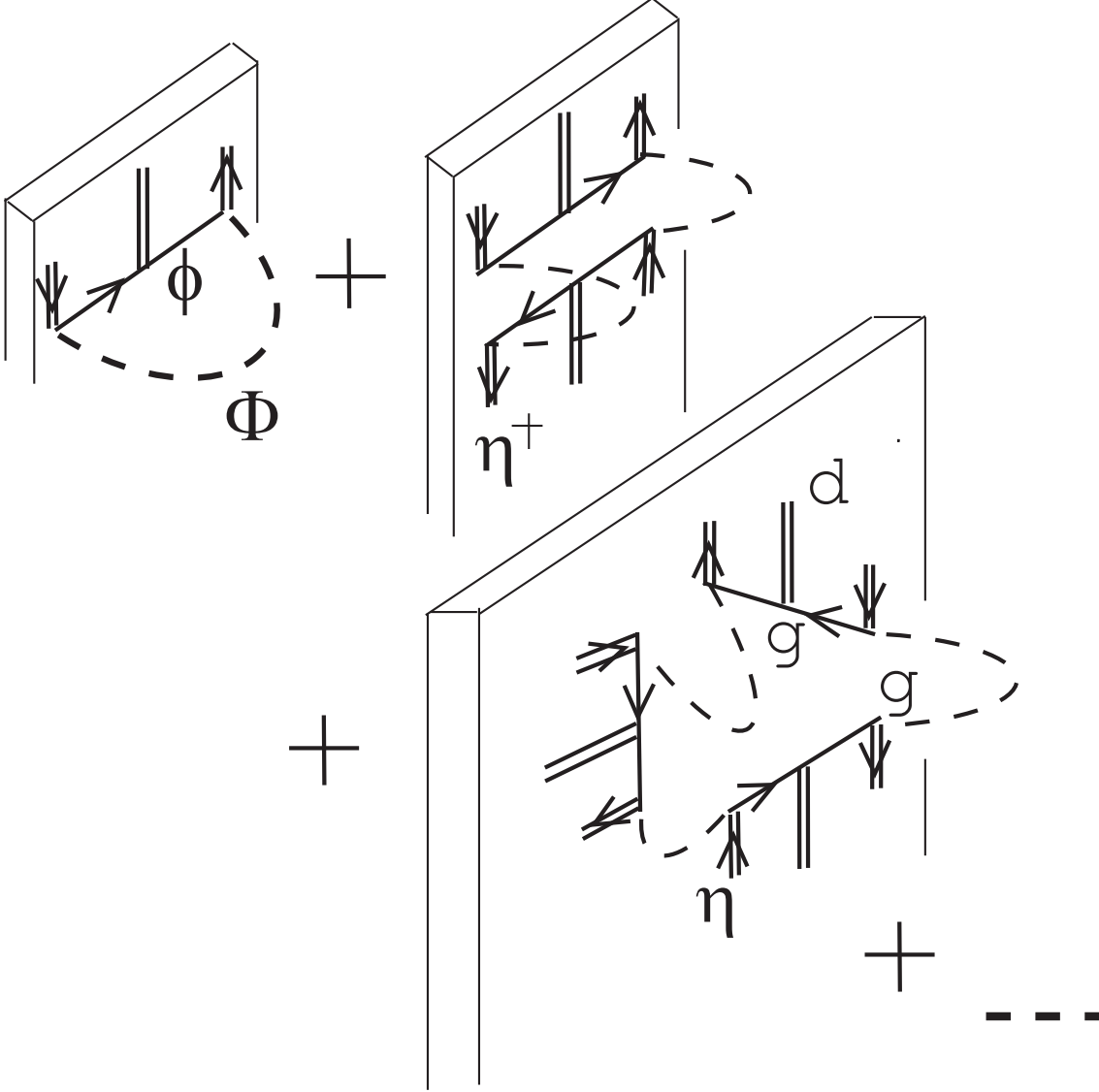


Figure 9: The diagrams contributing to the effective potential at the full (w.r.t. g) 1-loop level. The dotted lines represent quantum propagation of the bulk scalar Φ , the directed single lines represent that of the boundary (4D) scalars ϕ, ϕ^\dagger . The double lines represent boundary background fields η, η^\dagger and d . The "wall" represents the $x^5 = 0$ brane. The corresponding eigenvalues are given by (89). This figure is the bulk-boundary generalization of FIG.2 of ref.[9].

8 Conclusion

We have analyzed the quantum structure of a bulk-boundary system by taking the example of the Mirabelli-Peskin model. The analysis is newly formulated by the background field method. Feynman rules for the perturbative calculation are presented. We focus on the (1-loop) effective potential and the vacuum energy. It is confirmed that the singular terms well behave with the Kaluza-Klein modes summation. The whole effect can be regarded as some *deformation* of the 4D quantization. Its 4D reduction by $l \rightarrow 0$ is confirmed in the renormalization group calculation. The characteristic relation among the 4D-momentum \bar{k} , UV-cutoff Λ , and the IR-cutoff(S^1 radius) l appears. It comes from the requirement to escape from the linear divergence. The relation is the same one as in the lattice domain wall fermion. In addition to the bulk scalar Φ , the extra component of the bulk vector A^5 plays an important role in determining the vacuum. Especially their localized configurations are exploited. In the treatment, the vacuum is generalized in the sense that scalars may depend on the extra coordinate x^5 . In the intermediate stage, we have obtained a new type Casimir energy in addition to the ordinary type by Appelquist and Chodos. The obtained result of the effective potential includes the bulk-boundary generalization of the Coleman-Weinberg's case.

We hope the present analysis advances further development of the brane world physics.

Acknowledgment

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9 Appendix A: Effective Potential of Super QED

Now we consider the super QED. The action is most concisely described by one vector

superfield V and two chiral ones S (charge g) and R (charge $-g$):

$$\begin{aligned}
S &= \int d^4x d^2\theta d^2\bar{\theta} (\bar{S} e^{gV} S + \bar{R} e^{-gV} R) + \left\{ \int d^4x d^2\theta \left(\frac{1}{4} W W + m S R \right) + \text{h.c.} \right\} , \\
W^\alpha &= -\frac{1}{4} \bar{D}^2 D^\alpha V , \quad V = -\theta \sigma^m \bar{\theta} v_m + i \theta \theta \bar{\theta} \bar{\lambda} - i \bar{\theta} \bar{\theta} \theta \lambda + \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} D , \\
S &= A_S(y) + \sqrt{2} \theta \psi_S(y) + \theta \theta F_S(y) , \quad R = A_R(y) + \sqrt{2} \theta \psi_R(y) + \theta \theta F_R(y) , \\
y^m &= x^m + i \theta \sigma^m \bar{\theta} . \quad (91)
\end{aligned}$$

We focus on the scalar sector of the effective potential, based on the following points: 1) Lorentz invariance of the vacuum, 2) the 1-loop contribution from the non-scalar fields (spinors, vectors) can be recovered by taking "supersymmetric boundary condition". We put the condition.

$$v_m = 0 , \quad \lambda = \bar{\lambda} = 0 , \quad \psi_S = \psi_R = 0 . \quad (92)$$

Then the Lagrangian of the super QED reduces to the simple form.

$$\begin{aligned}
\mathcal{L}[A_S, F_S; A_R, F_R; D : m, g] &= \\
&\{ \bar{A}_S \partial_m \partial^m A_S + \bar{F}_S F_S + m(A_R F_S + \bar{A}_R \bar{F}_S) + R \leftrightarrow S \} \\
&+ \left\{ \frac{1}{2} D^2 + \frac{1}{2} g D (\bar{A}_S A_S - \bar{A}_R A_R) \right\} . \quad (93)
\end{aligned}$$

Now we expand all scalar fields around the background *constants* (a_S, f_S, \dots) .

$$A_S \rightarrow a_S + A_S , \quad F_S \rightarrow f_S + F_S , \quad D \rightarrow d + D , \quad \bar{A}_S \rightarrow \bar{a}_S + \bar{A}_S , \quad \bar{F} \rightarrow \bar{f}_S + \bar{F}_S , \quad (94)$$

and similarly for $A_R, \bar{A}_R, F_R, \bar{F}_R$.

The effective potential $V^{eff}[a, \bar{a}, f, \bar{f}, d]$ is defined as

$$\begin{aligned}
&\exp\left\{-i \int d^4x V^{eff}[a, \bar{a}, f, \bar{f}, d]\right\} = \int (\mathcal{D} A \mathcal{D} F \mathcal{D} \bar{A} \mathcal{D} \bar{F})_{S,R} \mathcal{D} D \\
&\times \exp i \int d^4x \left\{ \mathcal{L}[a + A, \bar{a} + \bar{A}, f + F, \bar{f} + \bar{F}, d + D] - \frac{\delta \mathcal{L}}{\delta \Phi^I} \Big|_b \Phi^I \right\} \\
&= \exp i \left\{ - \int d^4x V_0^{eff} \right\} \times \int (\mathcal{D} A \mathcal{D} F \mathcal{D} \bar{A} \mathcal{D} \bar{F})_{S,R} \mathcal{D} D \\
&\times \exp i \int d^4x \left\{ \mathcal{L}_2 + (\text{quantum field})^3 \text{ and higher-order} \right\} , \quad (95)
\end{aligned}$$

where $(\Phi^I) \equiv (A_{S,R}, F_{S,R}, \bar{A}_{S,R}, \bar{F}_{S,R}, D)$ are treated as the quantum fields and $(\Phi^I)|_b \equiv (a_{S,R}, f_{S,R}, \bar{a}_{S,R}, \bar{f}_{S,R}, d)$ are as the background fields. $-V_0^{eff}$ is the tree (zero-th order) part and is given below. \mathcal{L}_2 is the quadratic part and will be given in (100). The zero-th order is

$$\mathcal{L}_0 = -V_0^{eff} = \{ \bar{f}_S f_S + m(a_R f_S + \bar{a}_R \bar{f}_S) + R \leftrightarrow S \} + \frac{1}{2} d^2 + \frac{g}{2} d(\bar{a}_S a_S - \bar{a}_R a_R) . \quad (96)$$

The first order is given by

$$\mathcal{L}_1 = \frac{\delta \mathcal{L}}{\delta \Phi^I} \Big|_b \Phi^I = \{ \bar{F}_S(f_S + m\bar{a}_R) + (\bar{f}_S + ma_R)F_S + R \leftrightarrow S \} \\ + \{ A_R(mf_S - \frac{g}{2}d\bar{a}_R) + A_S(mf_R + \frac{g}{2}d\bar{a}_S) + \text{h.c.} \} + D\{d + \frac{g}{2}(\bar{a}_S a_S - \bar{a}_R a_R)\} . \quad (97)$$

There is a special choice of the background constants, the *on-shell condition*:

$$f_S + m\bar{a}_R = 0 \quad , \quad f_R + m\bar{a}_S = 0 \quad , \quad mf_S - \frac{g}{2}d\bar{a}_R = 0 \quad , \quad mf_R + \frac{g}{2}d\bar{a}_S = 0 \quad , \\ d + \frac{g}{2}(\bar{a}_S a_S - \bar{a}_R a_R) = 0 \quad , \quad (98)$$

and their complex conjugate. This is the solution of the field equation: $\mathcal{L}_1 = 0$. When the background constants satisfy the above equations, the tree effective potential V_0^{eff} takes

$$V_0^{eff}|_{\text{on-shell}} = \bar{f}_S f_S + \bar{f}_R f_R + \frac{1}{2}d^2 \geq 0 \quad . \quad (99)$$

From the results (98) and (99), the (classical) vacuum is given by the solution: $f_S = f_R = 0$, $d = 0$, $a_R = a_S = 0$ where $m \neq 0$ is assumed. In the following analysis, however, we consider the general case of the background constants. (We do *not* require the on-shell condition: $\mathcal{L}_1 = 0$.)

The second order part \mathcal{L}_2 is given by taking the quadratic terms with respect to the quantum fields $(A_{S,R}, F_{S,R}, \bar{A}_{S,R}, \bar{F}_{S,R}, D)$. \mathcal{L}_2 can be expressed in the following form, where F, \bar{F} -involved terms are separated.²⁶

$$\mathcal{L}_2 = \frac{1}{2} \begin{pmatrix} \bar{A}_S & A_S & \bar{A}_R & A_R & D \end{pmatrix} \mathbf{A} \begin{pmatrix} A_S \\ \bar{A}_S \\ A_R \\ \bar{A}_R \\ D \end{pmatrix} + \{ \bar{F}_S F_S + m(A_R F_S + \bar{A}_R \bar{F}_S) + R \leftrightarrow S \} , \\ \mathbf{A} = \begin{pmatrix} \square + dg/2 & 0 & 0 & 0 & ga_S/2 \\ 0 & \square + dg/2 & 0 & 0 & g\bar{a}_S/2 \\ 0 & 0 & \square - dg/2 & 0 & -ga_R/2 \\ 0 & 0 & 0 & \square - dg/2 & -g\bar{a}_R/2 \\ g\bar{a}_S/2 & ga_S/2 & -g\bar{a}_R/2 & -ga_R/2 & 1 \end{pmatrix} \quad (100)$$

The above matrix \mathbf{A} is the same as that in Ref.[17].²⁷ Integrating out all auxiliary fields $D, F_S, F_R, \bar{F}_S, \bar{F}_R$ using "squaring equations":

$$\left\{ \frac{1}{2}g(a_S \bar{A}_S - a_R \bar{A}_R)D + \text{c.c.} \right\} + \frac{1}{2}D^2$$

²⁶ f and \bar{f} disappear at this stage because the F and \bar{F} -auxiliary fields appear in (93) only as quadratic terms.

²⁷ In the paper, however, the contribution from F and \bar{F} is not taken into account.

$$\begin{aligned}
&= \frac{1}{2} \{ D + \frac{g}{2} (a_S \bar{A}_S - a_R \bar{A}_R + \text{c.c.}) \}^2 - \frac{g^2}{8} (a_S \bar{A}_S - a_R \bar{A}_R + \text{c.c.})^2 \quad , \\
&\quad \bar{F}_S F_S + m(A_R F_S + \bar{A}_R \bar{F}_S) + (S \leftrightarrow R) \\
&= (\bar{F}_S + m A_R)(F_S + m \bar{A}_R) - m^2 A_R \bar{A}_R + (S \leftrightarrow R) \quad , \quad (101)
\end{aligned}$$

\mathcal{L}_2 reduces to

$$\begin{aligned}
\mathcal{L}'_2 &= \bar{A}_S \square A_S + \bar{A}_R \square A_R + \frac{1}{2} \begin{pmatrix} \bar{A}_S & A_S & \bar{A}_R & A_R \end{pmatrix} \mathbf{M} \begin{pmatrix} A_S \\ \bar{A}_S \\ A_R \\ \bar{A}_R \end{pmatrix} \quad , \quad \square = \partial_m \partial^m \quad , \\
\mathbf{M} &= \begin{pmatrix} -Ga_S \bar{a}_S + \tilde{d} - m^2 & -G\bar{a}_S \bar{a}_S & Ga_R \bar{a}_S & G\bar{a}_R \bar{a}_S \\ -Ga_S a_S & -G\bar{a}_S a_S + \tilde{d} - m^2 & Ga_R a_S & G\bar{a}_R a_S \\ Ga_S \bar{a}_R & G\bar{a}_S \bar{a}_R & -Ga_R \bar{a}_R - \tilde{d} - m^2 & -G\bar{a}_R \bar{a}_R \\ Ga_S a_R & G\bar{a}_S a_R & -Ga_R a_R & -G\bar{a}_R a_R - \tilde{d} - m^2 \end{pmatrix} . \quad (102)
\end{aligned}$$

where $G \equiv g^2/4$, $\tilde{d} \equiv dg/2$. The four eigenvalues of \mathbf{M} are obtained as

$$\begin{aligned}
\lambda_1 &= \tilde{d} - m^2 = \frac{g}{2} d - m^2 \quad , \quad \lambda_2 = \tilde{d} - m^2 - 2Ga_S \bar{a}_S = \frac{g}{2} d - m^2 - \frac{g^2}{2} a_S \bar{a}_S \quad , \\
\lambda_3 &= -\tilde{d} - m^2 = -\frac{g}{2} d - m^2 \quad , \quad \lambda_4 = -\tilde{d} - m^2 - 2Ga_R \bar{a}_R = -\frac{g}{2} d - m^2 - \frac{g^2}{2} a_R \bar{a}_R \quad (103)
\end{aligned}$$

Then the 1-loop contribution is given as

$$\begin{aligned}
&\int (\mathcal{D}A \mathcal{D}\bar{A})_{S,R} \exp i \int d^4 x \mathcal{L}'_2 \\
&= [\det(\square + \lambda_1)(\square + \lambda_2)(\square + \lambda_3)(\square + \lambda_4)]^{-\frac{1}{2}} = \exp -i \int d^4 x V_{1-loop}^{eff} \quad , \\
&V_{1-loop}^{eff} = \frac{1}{2} \text{Tr} \sum_{i=1}^4 \ln(\square + \lambda_i) \quad . \quad (104)
\end{aligned}$$

Normalizing V_{1-loop}^{eff} at $d = 0$, from the requirement of the *supersymmetric boundary condition* (Sect.2), we finally obtain

$$\begin{aligned}
V_{1-loop}^{eff} - V_{1-loop}^{eff}|_{d=0} &= \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \left\{ \ln \left(1 - \left(\frac{g}{2}\right)^2 \frac{d^2}{(k^2 + m^2)^2} \right) \right. \\
&\quad + \ln \left(1 - \left(\frac{g}{2}\right)^2 \frac{d^2}{(k^2 + m^2 + g^2 a_S \bar{a}_S/2)(k^2 + m^2 + g^2 a_R \bar{a}_R/2)} \right. \\
&\quad \left. \left. + \frac{g}{2} \frac{g^2}{2} \frac{d(a_S \bar{a}_S - a_R \bar{a}_R)}{(k^2 + m^2 + g^2 a_S \bar{a}_S/2)(k^2 + m^2 + g^2 a_R \bar{a}_R/2)} \right) \right\} \\
&\approx \left(-\frac{g^4}{4} d^2 + \frac{g^3}{8} d(a_S \bar{a}_S - a_R \bar{a}_R) \right) \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + m^2)^2} + O(g^4) \quad . \quad (105)
\end{aligned}$$

The last approximate form is *logarithmically* divergent. We introduce a counterterm $\Delta\mathcal{L}$ as in the following form.

$$\begin{aligned} V_R &= V_0^{eff} + V_{1-loop}^{eff} - V_{1-loop}^{eff}|_{d=0} - \Delta\mathcal{L} \quad , \\ V_0^{eff} &= -\frac{1}{2}d^2 - \frac{1}{2}gd(\bar{a}_S a_S - \bar{a}_R a_R) + \cdots \quad , \\ \Delta\mathcal{L} &= \frac{1}{2}\Delta Z d^2 + \frac{1}{2}\Delta g d(\bar{a}_S a_S - \bar{a}_R a_R) \quad , \end{aligned} \quad (106)$$

where V_0^{eff} is the tree part of the potential (96). $Z = 1 + \Delta Z$ and $g_b = g + \Delta g$ are the *wave-function renormalization* constant of D and the *bare coupling constant*, respectively. We fix ΔZ and Δg by imposing the following *renormalization condition*.

$$\begin{aligned} \left. \frac{dV_R}{d(d^2)} \right|_{d=0, a=\bar{a}=0} &= -\frac{1}{2} \quad , \\ \left. \frac{dV_R}{d[\bar{a}_S a_S - \bar{a}_R a_R]} \right|_{d=0, a=\bar{a}=0} &= -\frac{1}{2}g \quad . \end{aligned} \quad (107)$$

Hence ΔZ and Δg are fixed as

$$\begin{aligned} \Delta Z &= -\frac{g^2}{2} \int_{|k| \leq \Lambda} \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + m^2)^2} = \left(-\frac{g^2}{2}\right) \frac{-1}{16\pi^2} \ln \frac{\Lambda^2}{m^2} \quad , \\ \Delta g &= \frac{g^3}{4} \int_{|k| \leq \Lambda} \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + m^2)^2} = \left(\frac{g^3}{4}\right) \frac{-1}{16\pi^2} \ln \frac{\Lambda^2}{m^2} \quad , \end{aligned} \quad (108)$$

where Λ is the momentum cutoff, $|k^2| \leq \Lambda^2$. Then the β -function of the coupling and the anomalous dimension γ of the D field are given as

$$\begin{aligned} g_b &= g + \Delta g = g \left(1 - \frac{g^2}{4} \frac{1}{16\pi^2} \ln \frac{\Lambda^2}{m^2}\right) \quad , \\ 0 &\equiv \frac{d}{d(\ln \Lambda)} \ln g_b = \frac{d \ln g}{d(\ln \Lambda)} (1 + O(g^2)) - \frac{g^2}{2} \frac{1}{16\pi^2} + O(g^2) \quad , \\ \beta(g) &\equiv \frac{1}{g} \frac{dg}{d \ln \Lambda} = \frac{g^2}{2} \frac{1}{16\pi^2} \quad , \\ Z &= 1 + \Delta Z \quad , \quad \gamma = \frac{\partial}{\partial \ln \Lambda} \ln Z = \frac{g^2}{16\pi^2} + O(g^4) \quad . \end{aligned} \quad (109)$$

Finally the on-shell potential is obtained as

$$\begin{aligned} V_R^{1-loop} &\equiv V_{1-loop}^{eff} - V_{1-loop}^{eff}|_{d=0} - \Delta\mathcal{L} \quad , \\ V_R|_{on-shell} &= (V_0^{eff} + V_R^{1-loop})|_{on-shell} \quad , \\ V_0^{eff}|_{on-shell} &= \bar{f}_S f_S + \bar{f}_R f_R + \frac{1}{2}d^2 \quad (\text{eq. (99)}) \quad , \\ V_R^{1-loop}|_{on-shell} &= \frac{1}{64\pi^2} \left[-g^2 d^2 + (m^4 + \frac{g^2}{4} d^2) \ln(1 + \frac{g}{2m^2} d) (1 - \frac{g}{2m^2} d) \right] \end{aligned}$$

$$\begin{aligned}
& -gm^2d \ln \frac{1 - \frac{g}{2m^2}d}{1 + \frac{g}{2m^2}d} - m^4(1 + \frac{g^2}{2m^4}\bar{f}_S f_S)^2 \ln \frac{1 + \frac{g^2}{2m^4}\bar{f}_S f_S}{1 + \frac{g^2}{2m^4}\bar{f}_S f_S + \frac{g}{2m^2}d} \\
& - m^4(1 + \frac{g^2}{2m^4}\bar{f}_R f_R)^2 \ln \frac{1 + \frac{g^2}{2m^4}\bar{f}_R f_R}{1 + \frac{g^2}{2m^4}\bar{f}_R f_R - \frac{g}{2m^2}d} \\
& + \frac{g^2}{4}d^2 \ln \left\{ (1 + \frac{g^2}{2m^4}\bar{f}_S f_S + \frac{g}{2m^2}d)(1 + \frac{g^2}{2m^4}\bar{f}_R f_R - \frac{g}{2m^2}d) \right\} \\
& + gm^2d(1 + \frac{g^2}{2m^4}\bar{f}_S f_S) \ln(1 + \frac{g^2}{2m^4}\bar{f}_S f_S + \frac{g}{2m^2}d) \\
& - gm^2d(1 + \frac{g^2}{2m^4}\bar{f}_R f_R) \ln(1 + \frac{g^2}{2m^4}\bar{f}_R f_R - \frac{g}{2m^2}d) \Big] \quad .(110)
\end{aligned}$$

For the case: $m = 1, \bar{f}_S f_S = \bar{f}_R f_R \equiv \bar{f}f$; the above potentials, $V_0^{eff}|_{\text{on-shell}}$ and $V_R^{1-loop}|_{\text{on-shell}}$, are depicted in Fig.10. The shape of the 1-loop correction is *not* the Coleman-Weinberg type. Positive definiteness is preserved after the renormalization. The potential minimum does not change. The minimum ($f_S = f_R = 0, d = 0, a_S = a_R = 0$) corresponds to the SUSY invariant vacuum. This shows a characteristic feature of the SUSY theory, that is, it is stable against the quantum effect. The form of the potential V_R does not essentially change from the tree one (99). The stableness of the vacuum was already pointed out, in the counter-term calculation, by Barbieri et al[21]. We confirm it by the explicit form of the renormalized potential.

10 Appendix B: Eigenvalues of Mass Matrix \mathcal{M} and Effective Potential of the Mirabelli-Peskin Model

The effective potential of the present bulk-boundary model can be obtained from the eigenvalues of the mass matrix \mathcal{M} of (79) and (80). It is made of three field values $\eta, \eta^\dagger, d_\alpha$, two wall "heights", $\bar{a}_\alpha, \bar{\varphi}_\alpha$, the gauge coupling, g and the boundary parameter l . The general case is hard to analyze explicitly. Here we consider two interesting "sections": A) $\eta = 0, \eta^\dagger = 0$ (bulk-boundary decoupled case); B) $\bar{\varphi} = 0, \bar{a} = 0$ (bulk-boundary coupled case).

10.1 Effective Potential From Matter Field Vanishing Point — Case A) $\eta = 0, \eta^\dagger = 0$ —

In this configuration, the interaction term $g\mathcal{D}_\alpha\phi^\dagger T^\alpha\phi$ of (32) does not contribute to the bulk-boundary loop. The bulk and boundary are *decoupled* in the quantum fluctuation. It turns out, however, that the *renormalization* procedure to

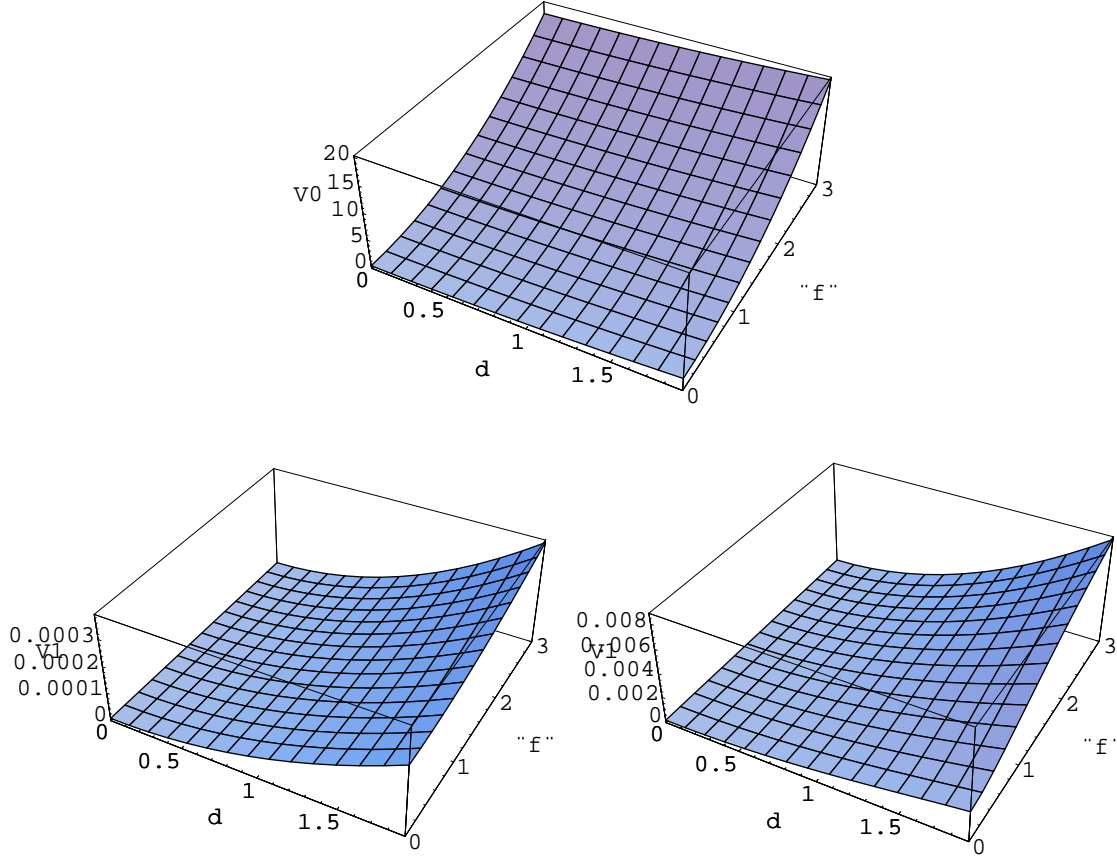


Figure 10: The effective potential of SQED(110) for the case: $m = 1, \bar{f}_S f_S = \bar{f}_R f_R \equiv \bar{f}f$. The tree part ($V_0^{eff}|_{\text{on-shell}} \equiv V_0$, above) and the 1-loop correction part ($V_R^{1-loop}|_{\text{on-shell}} \equiv V_1$, below) are depicted for $g = 0.3$ (left), $g = 1$ (right). The axis-ranges are $0 \leq d \leq 1.9, 0 \leq |f| \equiv \sqrt{\bar{f}f} \leq 3$.

deal with the divergences due to the boundary-loop makes connection between the bulk and boundary. (See the explanation below (117).) \mathcal{M} has the following form:

$$\begin{pmatrix} \begin{pmatrix} \mathcal{M}_{\phi^\dagger\phi} & 0 \\ 0 & \mathcal{M}_{\phi\phi^\dagger} \end{pmatrix}_{\alpha'\beta'} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{\alpha'n\beta} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{m\alpha\beta'} & \begin{pmatrix} \mathcal{M}_{\Phi\Phi} & \mathcal{M}_{\Phi A} \\ \mathcal{M}_{A\Phi} & \mathcal{M}_{AA} \end{pmatrix}_{m\alpha n\beta} \end{pmatrix}. \quad (111)$$

The components are given, from (80), as

$$\begin{aligned} \mathcal{M}_{\phi_{\alpha'}^\dagger\phi_{\beta'}} &= \partial^2\delta_{\alpha'\beta'} + g d_\gamma(T^\gamma)_{\alpha'\beta'} \quad , \\ \mathcal{M}_{\phi_{\alpha'}\phi_{\beta'}^\dagger} &= \partial^2\delta_{\alpha'\beta'} + g d_\gamma(T^\gamma)_{\beta'\alpha'} \quad , \\ \mathcal{M}_{\Phi_{m\alpha}\Phi_{n\beta}} &= -\{-\partial^2 + (\frac{n\pi}{l})^2\}\delta_{mn}\delta_{\alpha\beta} - g^2 f_{\alpha\delta\tau} f_{\beta\gamma\tau} \bar{a}_\delta \bar{a}_\gamma \delta_{mn} + \frac{4g}{l} f_{\alpha\beta\gamma} \bar{a}_\gamma m Q_{mn} \quad , \\ \mathcal{M}_{\Phi_{m\alpha}A_{n\beta}} &= g^2 f_{\alpha\beta\tau} f_{\gamma\delta\tau} \bar{a}_\gamma \bar{\varphi}_\delta \delta_{mn} - g^2 f_{\gamma\alpha\tau} f_{\beta\delta\tau} \bar{a}_\gamma \bar{\varphi}_\delta \delta_{mn} - \frac{2g}{l} f_{\alpha\beta\gamma} \bar{\varphi}_\gamma m Q_{mn} = M_{A_{n\beta}\Phi_{m\alpha}}, \\ \mathcal{M}_{A_{m\alpha}A_{n\beta}} &= -\{-\partial^2 + (\frac{n\pi}{l})^2\}\delta_{mn}\delta_{\alpha\beta} - g^2 f_{\alpha\gamma\tau} f_{\beta\delta\tau} \bar{\varphi}_\gamma \bar{\varphi}_\delta \delta_{mn} \quad (112) \end{aligned}$$

where the integer indices m, n run from 1 to ∞ . Q_{mn} is defined in (71). The singular terms, $\delta(0)$ -terms, disappear. The bulk and the boundary are decoupled, hence the eigen values can be obtained separately.

For simplicity we take $SU(2)$ as the gauge group G ($f_{\alpha\beta\gamma} = \epsilon^{\alpha\beta\gamma}$) and the doublet representation for the matter fields $\phi_{\alpha'}$ ($\alpha' = 1, 2$).

$$T^\alpha = \frac{1}{2}\sigma^\alpha \quad , \quad [T^\alpha, T^\beta] = i\epsilon^{\alpha\beta\gamma}T^\gamma \quad , \quad \text{Tr}(T^\alpha T^\beta) = \frac{1}{2}\delta^{\alpha\beta} \quad , \quad (113)$$

where σ^α ($\alpha = 1, 2, 3$) is the Pauli sigma matrices, and $\epsilon^{123} = 1$.

(Ai) boundary part

The eigenvalues of

$$\left(\mathcal{M}_{\phi_{\alpha'}^\dagger\phi_{\beta'}}\right) = \begin{pmatrix} -k^2 + \frac{g}{2}d_3 & \frac{g}{2}(d_1 - id_2) \\ \frac{g}{2}(d_1 + id_2) & -k^2 - \frac{g}{2}d_3 \end{pmatrix} \quad , \quad (114)$$

are

$$\lambda_\pm = -k^2 \pm \frac{g}{2}\sqrt{d^2} \quad , \quad d^2 \equiv d_1^2 + d_2^2 + d_3^2 \quad , \quad (115)$$

The same ones are obtained for $(\mathcal{M}_{\phi_{\alpha'}\phi_{\beta'}^\dagger})$. The effective potential can be obtained as

$$\begin{aligned} V_{1-loop}^{eff} &= \frac{1}{2}\text{Tr} \ln(\lambda_+)^2(\lambda_-)^2 = \text{Tr} \ln\{(k^2)^2 - \frac{g^2}{4}d^2\} \\ &= \int \frac{d^4k}{(2\pi)^4} [\ln(k^2)^2 + \ln\{1 - \frac{g^2}{4}\frac{d^2}{(k^2)^2}\}] \quad . \end{aligned} \quad (116)$$

Compare this result with the super QED case (the first line of (105)). Taking the SUSY condition (see the explanation given above (12)), we reach the final answer.

$$\begin{aligned} V_{1-loop}^{eff} &= \int \frac{d^4 k}{(2\pi)^4} \ln \left\{ 1 - \frac{g^2}{4} \frac{d^2}{(k^2)^2} \right\} \\ &= -\frac{g^2}{4} \int \frac{d^4 k}{(2\pi)^4} \frac{d^2}{(k^2)^2} + O(g^4) \quad . \end{aligned} \quad (117)$$

The last approximated form corresponds to (c) of (48), and is *logarithmically* divergent. Because d_α is given by $d_\alpha = (\chi^3 - \partial_5 \varphi + g a_5 \times \varphi)_\alpha$, the UV divergence of (117) is renormalized by the *bulk* wave function of X^3 and Φ . Here the 4D world's connection to the bulk world appears. The quantum fluctuation within the boundary influence the bulk world through the renormalization. The boundary dynamics does *not* close within the brane. We do not touch on the renormalization of the bulk fields. After an appropriate renormalization, we expect the effective potential (117) leads to a similar potential to that given in App.A for the case $m = 0, \bar{f}f = 0$. (Note that the boundary theory treated in this section is chiral, whereas the Super QED treated in App.A is vector-like.) We may conclude that, in the vacuum specified by $\eta = 0, \eta^\dagger = 0$, the renormalization works well as far as the boundary world is concerned. The 4D theory is well-defined.

(Aii) Bulk Part

Let us evaluate the eigenvalues from the bulk part. Because it does not depend on d_α , this part does *not* contribute to the effective potential in the SUSY boundary condition. However it is important to see what terms are quantumly induced by the scalar fields. (Those terms are expected to be cancelled by the fermion and vector fields contribution.) The result depends on the "heights" of the 4D scalars, \bar{a} and $\bar{\varphi}$, in addition to the periodicity $2l$. Generally that part of the vacuum energy which depends on the boundary parameters is called "Casimir energy". We regard \bar{a} and $\bar{\varphi}$, besides l , as those parameters. They correspond to the brane tension and the brane thickness in the brane world. One of most important points of the brane model is how to treat the KK-modes. We can see such a point in this calculation.

The eigenvalue equation can be written as

$$\begin{pmatrix} \mathcal{M}_{\Phi_{m\alpha}\Phi_{n\beta}} & \mathcal{M}_{\Phi_{m\alpha}A_{n\beta}} \\ \mathcal{M}_{A_{m\alpha}\Phi_{n\beta}} & \mathcal{M}_{A_{m\alpha}A_{n\beta}} \end{pmatrix} \begin{pmatrix} \hat{\Phi}_{n\beta} \\ \hat{A}_{n\beta} \end{pmatrix} = \lambda \begin{pmatrix} \hat{\Phi}_{m\alpha} \\ \hat{A}_{m\alpha} \end{pmatrix} \quad . \quad (118)$$

From the symmetry, we can take the following general form as an eigen vector.

$$\begin{aligned} \hat{\Phi}_{n\beta} &= f_1(n)\bar{a}_\beta + f_2(n)\bar{\varphi}_\beta + f_3(n)f_{\beta\gamma\delta}\bar{a}_\gamma\bar{\varphi}_\delta \quad , \\ \hat{A}_{n\beta} &= g_1(n)\bar{a}_\beta + g_2(n)\bar{\varphi}_\beta + g_3(n)f_{\beta\gamma\delta}\bar{a}_\gamma\bar{\varphi}_\delta \quad , \end{aligned} \quad (119)$$

where $f_i(n)$ and $g_i(n)$ are scalar quantities (with respect to the internal group

transformation) which may depend on \bar{a} and $\bar{\varphi}$.²⁸ ²⁹ Through the above relation, the eigenvalue equation for $\hat{\Phi}$ and \hat{A} is replaced by that for $f_i(n)$ and $g_i(n)$. The eigenvalues are obtained from the zeros of the determinant of the following matrix.

	$f_1(n)$	$f_2(n)$	$f_3(n)$	$g_1(n)$	$g_2(n)$	$g_3(n)$
\bar{a}, m	a			c		
$\bar{\varphi}, m$						
$\bar{a} \times \bar{\varphi}, m$						
\bar{a}, m	d			b		
$\bar{\varphi}, m$						
$\bar{a} \times \bar{\varphi}, m$						

(120)

The above 4 matrices are given as follows.

The first row equation of (118), $\mathcal{M}_{\Phi_{m\alpha}\Phi_{n\beta}}\hat{\Phi}_{n\beta} + \mathcal{M}_{\Phi_{m\alpha}A_{n\beta}}\hat{A}_{n\beta} = \lambda\hat{\Phi}_{m\alpha}$, gives two matrices a and c as

$$a = \begin{pmatrix} (-\lambda - k^2 - (m\pi/l)^2)\delta_{mn} & g^2\bar{a} \cdot \bar{\varphi}\delta_{mn} & -4gmQ_{mn}\bar{a} \cdot \bar{\varphi}/l \\ 0 & (-\lambda - k^2 - (m\pi/l)^2 - g^2\bar{a}^2)\delta_{mn} & 4gmQ_{mn}\bar{a}^2/l \\ 0 & -4gmQ_{mn}/l^2 & (-\lambda - k^2 - (m\pi/l)^2 - g^2\bar{a}^2)\delta_{mn} \end{pmatrix},$$

$$c = \begin{pmatrix} 2g^2\bar{a} \cdot \bar{\varphi}\delta_{mn} & g^2\bar{\varphi}^2\delta_{mn} & 2gmQ_{mn}\bar{\varphi}^2/l \\ -2g^2\bar{a}^2\delta_{mn} & -g^2\bar{a} \cdot \bar{\varphi}\delta_{mn} & -2gmQ_{mn}\bar{a} \cdot \bar{\varphi}/l \\ -2gmQ_{mn}/l^4 & 0 & g^2\bar{a} \cdot \bar{\varphi}\delta_{mn} \end{pmatrix} \quad (121)$$

The second row equation of (118), $\mathcal{M}_{A_{m\alpha}\Phi_{n\beta}}\hat{\Phi}_{n\beta} + \mathcal{M}_{A_{m\alpha}A_{n\beta}}\hat{A}_{n\beta} = \lambda\hat{A}_{m\alpha}$, gives two matrices d and b as

$$d = \begin{pmatrix} -g^2\bar{a} \cdot \bar{\varphi}\delta_{mn} & -2g^2\bar{\varphi}^2\delta_{mn} & -2gnQ_{nm}\bar{\varphi}^2/l \\ g^2\bar{a}^2\delta_{mn} & 2g^2\bar{a} \cdot \bar{\varphi}\delta_{mn} & 2gnQ_{nm}\bar{a} \cdot \bar{\varphi}/l \\ 2gnQ_{nm}/l^4 & 0 & g^2\bar{a} \cdot \bar{\varphi}\delta_{mn} \end{pmatrix},$$

$$b = \begin{pmatrix} (-\lambda - k^2 - (m\pi/l)^2 - g^2\bar{\varphi}^2)\delta_{mn} & 0 & 0 \\ g^2\bar{a} \cdot \bar{\varphi}\delta_{mn} & (-\lambda - k^2 - (m\pi/l)^2)\delta_{mn} & 0 \\ 0 & 0 & (-\lambda - k^2 - (m\pi/l)^2 - g^2\bar{\varphi}^2)\delta_{mn} \end{pmatrix} \quad (122)$$

For convenience, let us introduce two quantities x_m, y_m ;

$$x_m \equiv -\lambda - k^2 - (m\pi/l)^2 - g^2\bar{\varphi}^2, \quad y_m \equiv -\lambda - k^2 - (m\pi/l)^2, \\ x_m - y_m = -g^2\bar{\varphi}^2. \quad (123)$$

Then the following relations are obtained.

$$b = \begin{pmatrix} x_m\delta_{mn} & 0 & 0 \\ g^2\bar{a} \cdot \bar{\varphi}\delta_{mn} & y_m\delta_{mn} & 0 \\ 0 & 0 & x_m\delta_{mn} \end{pmatrix},$$

²⁸The physical dimensions of the "coefficients" functions are as follows; $[f_1] = [f_2] = M^{-1/2}$, $[f_3] = M^{-2}$, $[g_1] = [g_2] = M^{-1/2}$, $[g_3] = M^{-2}$.

²⁹The change of the vector space of the eigen functions makes the number of eigenvalues change. We can, however, choose proper values from the consistency with the perturbative results of Sec.5.

$$\mathbf{b}^{-1} = \begin{pmatrix} \frac{1}{x_m} \delta_{mn} & 0 & 0 \\ -\frac{g^2 \bar{a} \cdot \bar{\varphi}}{x_m y_m} \delta_{mn} & \frac{1}{y_m} \delta_{mn} & 0 \\ 0 & 0 & \frac{1}{x_m} \delta_{mn} \end{pmatrix},$$

$$\det \mathbf{b} = \prod_{m=1}^{\infty} (x_m^2 y_m). \quad (124)$$

Using a useful formula about general matrices $\tilde{a}, \tilde{b}, \tilde{c}$, and \tilde{d} ($\det \tilde{b} \neq 0, \det \tilde{a} \neq 0$);

$$\det \begin{pmatrix} \tilde{a} & \tilde{c} \\ \tilde{d} & \tilde{b} \end{pmatrix} = \det(\tilde{a} - \tilde{c} \tilde{b}^{-1} \tilde{d}) \det \tilde{b} = \det \tilde{a} \det(\tilde{b} - \tilde{d} \tilde{a}^{-1} \tilde{c}) \quad (125)$$

we can decompose the determinant of the matrix (120). The components of the matrix $\mathcal{A} \equiv a - cb^{-1}d$ are explicitly written as

$$\mathcal{A} = \begin{array}{c|ccc} & f_1(n) & f_2(n) & f_3(n) \\ \hline \bar{a}, m & \mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{A}_{13} \\ \bar{\varphi}, m & \mathcal{A}_{21} & \mathcal{A}_{22} & \mathcal{A}_{23} \\ \bar{a} \times \bar{\varphi}, m & \mathcal{A}_{31} & \mathcal{A}_{32} & \mathcal{A}_{33} \end{array},$$

$$(\mathcal{A}_{11})_{mn} = y_m \delta_{mn} - g^4 \left\{ (\bar{a} \cdot \bar{\varphi})^2 \frac{1}{x_m} (-2 + \frac{g^2 \bar{\varphi}^2}{y_m}) + \frac{\bar{\varphi}^2 \bar{a}^2}{y_m} \right\} \delta_{mn} - 4 \frac{g^2 \bar{\varphi}^2}{l^2} mn \sum_j \frac{Q_{mj} Q_{nj}}{x_j},$$

$$(\mathcal{A}_{12})_{mn} = g^2 \bar{a} \cdot \bar{\varphi} \left\{ 1 + 2g^2 \frac{\bar{\varphi}^2}{x_m} \right\} \delta_{mn} \quad , \quad (\mathcal{A}_{13})_{mn} = -\frac{\bar{a} \cdot \bar{\varphi}}{l} Q_{mn} \left\{ 4gm + 2g^3 \bar{\varphi}^2 \left(\frac{n}{x_m} + \frac{m}{x_n} \right) \right\},$$

$$(\mathcal{A}_{21})_{mn} = -g^4 \frac{\bar{a} \cdot \bar{\varphi}}{x_m} \left\{ \bar{a}^2 + \frac{g^2}{y_m} (\bar{a}^2 \bar{\varphi}^2 - (\bar{a} \cdot \bar{\varphi})^2) \right\} \delta_{mn} + 4 \frac{g^2 \bar{a} \cdot \bar{\varphi}}{l^2} mn \sum_j \frac{Q_{mj} Q_{nj}}{x_j},$$

$$(\mathcal{A}_{22})_{mn} = \left\{ y_m - g^2 \bar{a}^2 - \frac{2g^4}{x_m} (2\bar{a}^2 \bar{\varphi}^2 - (\bar{a} \cdot \bar{\varphi})^2) \right\} \delta_{mn} \quad ,$$

$$(\mathcal{A}_{23})_{mn} = \frac{Q_{mn}}{l} \left\{ 4g\bar{a}^2 m + 2g^3 \left(\frac{n}{x_m} - \frac{m}{x_n} \right) (\bar{a}^2 \bar{\varphi}^2 - (\bar{a} \cdot \bar{\varphi})^2) \right\} \quad ,$$

$$(\mathcal{A}_{31})_{mn} = 2 \frac{g^3 \bar{a} \cdot \bar{\varphi}}{l^4} Q_{mn} \left(\frac{n}{x_m} - \frac{m}{x_n} \right) \quad , \quad (\mathcal{A}_{32})_{mn} = -\frac{Q_{mn}}{l^4} \left(4gm + 4g^3 \bar{\varphi}^2 \frac{m}{x_n} \right) \quad ,$$

$$(\mathcal{A}_{33})_{mn} = \left\{ y_m - g^2 \bar{a}^2 - g^4 \frac{(\bar{a} \cdot \bar{\varphi})^2}{x_m} \right\} \delta_{mn} - 4 \frac{g^2 \bar{\varphi}^2}{l^2} mn \sum_j \frac{Q_{mj} Q_{nj}}{x_j} \quad (126)$$

where the repeated integer suffixes do *not* mean the summation.³⁰ The summation should be taken only where the symbol \sum_j appears.

Let us evaluate the eigenvalues λ from the zeros of $\det \mathcal{A}$. General case is technically difficult. We consider the following special cases.

We consider the following limit:

$$\hat{g}^2 \equiv \frac{g^2}{l} = \text{fixed} \ll 1 \quad , \quad \hat{a} = \sqrt{l} \bar{a} = \text{fixed} \quad , \quad \hat{\varphi} = \sqrt{l} \bar{\varphi} = \text{fixed} \quad ,$$

³⁰Because of this, the product $Q_{mn}(\frac{n}{x_m} + \frac{m}{x_n})$ appearing in $(\mathcal{A}_{13})_{mn}$ of (126), does not vanish in spite of the antisymmetry of Q_{mn} .

$$l \rightarrow \infty \quad . \quad (127)$$

This is the situation where the circle is large compared with the inverse of the domain wall height. (\hat{a} and $\hat{\varphi}$ have the dimension of M .) Then the elements of \mathcal{A} reduce to

$$\begin{aligned} (\mathcal{A}_{11})_{mn} &= y\delta_{mn} - \hat{g}^4 \left\{ (\hat{a} \cdot \hat{\varphi})^2 \frac{1}{x} (-2 + \frac{\hat{g}^2 \hat{\varphi}^2}{y}) + \frac{\hat{\varphi}^2 \hat{a}^2}{y} \right\} \delta_{mn} \quad , \\ (\mathcal{A}_{12})_{mn} &= \hat{g}^2 \hat{a} \cdot \hat{\varphi} \left\{ 1 + 2\hat{g}^2 \frac{\hat{\varphi}^2}{x} \right\} \delta_{mn} \quad , \quad (\mathcal{A}_{13})_{mn} = 0 \quad , \\ (\mathcal{A}_{21})_{mn} &= -\hat{g}^4 \frac{\hat{a} \cdot \hat{\varphi}}{x} \left\{ \hat{a}^2 + \frac{\hat{g}^2}{y} (\hat{a}^2 \hat{\varphi}^2 - (\hat{a} \cdot \hat{\varphi})^2) \right\} \delta_{mn} \quad , \\ (\mathcal{A}_{22})_{mn} &= \left\{ y - \hat{g}^2 \hat{a}^2 - \frac{2\hat{g}^4}{x} (2\hat{a}^2 \hat{\varphi}^2 - (\hat{a} \cdot \hat{\varphi})^2) \right\} \delta_{mn} \quad , \\ (\mathcal{A}_{23})_{mn} &= 0 \quad , \\ (\mathcal{A}_{31})_{mn} &= 0 \quad , \quad (\mathcal{A}_{32})_{mn} = 0 \quad , \\ (\mathcal{A}_{33})_{mn} &= \left\{ y - \hat{g}^2 \hat{a}^2 - \hat{g}^4 \frac{(\hat{a} \cdot \hat{\varphi})^2}{x} \right\} \delta_{mn} \quad , \end{aligned} \quad (128)$$

where $x \equiv -\lambda - k^2 - \hat{g}^2 \hat{\varphi}^2$, $y \equiv -\lambda - k^2$. We notice, in this limit, Q_{mn} -terms disappear. In the "propagator" terms x_m, y_m , KK-mass terms $m^2 \pi^2 / l^2$ disappear. All KK-modes equally contribute to the vacuum energy. The condition $0 = \det \mathcal{A} = \det \mathcal{A}_{33} \det \mathcal{A}_{22} \det (\mathcal{A}_{11} - \mathcal{A}_{12} \mathcal{A}_{22}^{-1} \mathcal{A}_{21})$ gives us the following eigenvalues.

(i) $\det \mathcal{A}_{33} = 0$ gives two eigenvalues λ_1, λ_2 whose product is given by

$$\lambda_1 \lambda_2 = (k^2 + \hat{g}^2 \hat{\varphi}^2)(k^2 + \hat{g}^2 \hat{a}^2) - \hat{g}^4 (\hat{a} \cdot \hat{\varphi})^2 \quad . \quad (129)$$

(ii) $\det \mathcal{A}_{22} = 0$ gives λ_3, λ_4 whose product is

$$\lambda_3 \lambda_4 = (k^2)^2 + k^2 \hat{g}^2 (\hat{\varphi}^2 + \hat{a}^2) + \hat{g}^4 (-3\hat{\varphi}^2 \hat{a}^2 + (\hat{a} \cdot \hat{\varphi})^2) \quad . \quad (130)$$

(iii) $\det (\mathcal{A}_{11} - \mathcal{A}_{12} \mathcal{A}_{22}^{-1} \mathcal{A}_{21}) = 0$ gives $\lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9$ whose product is given by

$$\begin{aligned} \lambda_5 \lambda_6 \lambda_7 \lambda_8 \lambda_9 &= -(k^2 + \hat{g}^2 \hat{\varphi}^2) \{ (k^2)^2 + \hat{g}^4 ((\hat{a} \cdot \hat{\varphi})^2 - \hat{a}^2 \hat{\varphi}^2) \} \\ &\quad \{ (k^2)^2 + \hat{g}^2 k^2 (\hat{a}^2 + \hat{\varphi}^2) + 3\hat{g}^4 ((\hat{a} \cdot \hat{\varphi})^2 - \hat{a}^2 \hat{\varphi}^2) \} \quad . \end{aligned} \quad (131)$$

In particular, for the special case $\hat{a} = 0$, the nontrivial factor is only $k^2 + \hat{g}^2 \hat{\varphi}^2$. Hence each KK-mode equally contribute to the vacuum energy as

$$V_{1KK-mode}^{eff} \propto \int \frac{d^4 k}{(2\pi)^4} \ln \{ 1 + \hat{g}^2 \frac{\hat{\varphi}^2}{k^2} \} \quad . \quad (132)$$

This quantity is *quadratically* divergent. After an appropriate normalization, which we do not know precisely, the final form should become, based on the dimensional analysis, the following.

$$\frac{1}{l} V_{1-loop}^{eff} = \hat{g}^2 (c_1 \frac{\hat{\varphi}^2}{l^3} + c_2 \frac{\hat{a}^2}{l^3} + c_3 \frac{\hat{a} \cdot \hat{\varphi}}{l^3}) + O(\hat{g}^4) \quad , \quad (133)$$

where c_1, c_2 and c_3 are some finite constants. This is a *new* type Casimir energy. Comparing the ordinary one (137) explained soon, it is new in the following points: 1) it depends on the brane parameters $\hat{\varphi}$ and \hat{a} besides the extra-space size l ; 2) it depends on the gauge coupling; 3) it is proportional to $1/l^3$.

10.2 Effective Potential With No Brane Structure — Case (B) $\bar{\varphi} = 0, \bar{a} = 0$ —

Let us evaluate the case B), $\bar{\varphi} = 0, \bar{a} = 0$. In this case 5D vacuum does not have the brane structure. The situation is similar to the case of Appelquist and Chodos's work. The matrix \mathcal{M} has the form:

$$\left(\begin{array}{cc} \left(\begin{array}{cc} \mathcal{M}_{\phi^\dagger\phi} & \mathcal{M}_{\phi^\dagger\phi^\dagger} \\ \mathcal{M}_{\phi\phi} & \mathcal{M}_{\phi\phi^\dagger} \end{array} \right)_{\alpha'\beta'} & \left(\begin{array}{cc} \mathcal{M}_{\phi^\dagger\Phi} & 0 \\ \mathcal{M}_{\phi\Phi} & 0 \end{array} \right)_{\alpha'n\beta} \\ \left(\begin{array}{cc} \mathcal{M}_{\Phi\phi} & \mathcal{M}_{\Phi\phi^\dagger} \\ 0 & 0 \end{array} \right)_{m\alpha\beta'} & \left(\begin{array}{cc} \mathcal{M}_{\Phi\Phi} & 0 \\ 0 & \mathcal{M}_{AA} \end{array} \right)_{m\alpha n\beta} \end{array} \right), \quad (134)$$

where each component is described as

$$\begin{aligned} \mathcal{M}_{\phi_{\alpha'}^\dagger\phi_{\beta'}} &= \partial^2\delta_{\alpha'\beta'} + gd_\gamma(T^\gamma)_{\alpha'\beta'} - g^2\delta(0)(T^\gamma\eta)_{\alpha'}(\eta^\dagger T^\gamma)_{\beta'} \quad , \\ \mathcal{M}_{\phi_{\alpha'}^\dagger\phi_{\beta'}^\dagger} &= -g^2\delta(0)(T^\gamma\eta)_{\alpha'}(T^\gamma\eta)_{\beta'} \quad , \quad \mathcal{M}_{\phi_{\alpha'}\phi_{\beta'}} = -g^2\delta(0)(\eta^\dagger T^\gamma)_{\alpha'}(\eta^\dagger T^\gamma)_{\beta'} \quad , \\ \mathcal{M}_{\phi_{\alpha'}\phi_{\beta'}^\dagger} &= \partial^2\delta_{\alpha'\beta'} + gd_\gamma(T^\gamma)_{\beta'\alpha'} - g^2\delta(0)(\eta^\dagger T^\gamma)_{\alpha'}(T^\gamma\eta)_{\beta'} \\ \mathcal{M}_{\phi_{\alpha'}^\dagger\Phi_{n\beta}} &= -\frac{g}{\sqrt{l}}(T^\beta\eta)_{\alpha'}\frac{n\pi}{l} = \mathcal{M}_{\Phi_{n\beta}\phi_{\alpha'}^\dagger} \quad , \quad \mathcal{M}_{\phi_{\alpha'}\Phi_{n\beta}} = -\frac{g}{\sqrt{l}}(\eta^\dagger T^\beta)_{\alpha'}\frac{n\pi}{l} = \mathcal{M}_{\Phi_{n\beta}\phi_{\alpha'}} \quad , \\ \mathcal{M}_{\Phi_{m\alpha}\Phi_{n\beta}} &= -\{-\partial^2 + (\frac{n\pi}{l})^2\}\delta_{mn}\delta_{\alpha\beta} \quad , \\ \mathcal{M}_{A_{m\alpha}A_{n\beta}} &= -\{-\partial^2 + (\frac{n\pi}{l})^2\}\delta_{mn}\delta_{\alpha\beta} \quad (135) \end{aligned}$$

where the integer indices m and n run from 1 to ∞ . Q_{mn} -terms disappear. Let us find the eigenvalues of the above matrix.

\mathcal{M}_{AA} part is decoupled with others, hence the eigenvalues of the part is obtained as

$$\lambda_n = -k^2 - (\frac{n\pi}{l})^2 \quad , \quad n = 1, 2, 3, \dots \quad . \quad (136)$$

These correspond to the massive KK-modes of the fifth component of the bulk vector. The eigenvalues (136) and another ones (146) explained soon, have the same form as that appearing in the work by Appelquist and Chodos[7]. They contribute to the Casimir energy.

$$\frac{1}{l} V_{Casimir}^{eff} = \frac{\text{const}}{l^5} \quad . \quad (137)$$

The eigenvalue equation for the other parts can be written as

$$\mathcal{M}_1 \begin{pmatrix} \hat{\phi} \\ \hat{\phi}^\dagger \\ \hat{\Phi} \end{pmatrix} \equiv \begin{pmatrix} \begin{pmatrix} \mathcal{M}_{\phi^\dagger\phi} & \mathcal{M}_{\phi^\dagger\phi^\dagger} \\ \mathcal{M}_{\phi\phi} & \mathcal{M}_{\phi\phi^\dagger} \end{pmatrix}_{\alpha'\beta'} & \begin{pmatrix} \mathcal{M}_{\phi^\dagger\Phi} \\ \mathcal{M}_{\phi\Phi} \end{pmatrix}_{\alpha'n\beta} \\ \begin{pmatrix} \mathcal{M}_{\Phi\phi} & \mathcal{M}_{\Phi\phi^\dagger} \end{pmatrix}_{m\alpha\beta'} & \begin{pmatrix} \mathcal{M}_{\Phi\Phi} \end{pmatrix}_{m\alpha n\beta} \end{pmatrix} \begin{pmatrix} \hat{\phi}_{\beta'} \\ \hat{\phi}_{\beta'}^\dagger \\ \hat{\Phi}_{n\beta} \end{pmatrix} = \lambda \begin{pmatrix} \hat{\phi}_{\alpha'} \\ \hat{\phi}_{\alpha'}^\dagger \\ \hat{\Phi}_{m\alpha} \end{pmatrix} \quad (138)$$

We can take the following form as the eigen vector, from the transformation property.

$$\begin{aligned} \hat{\phi}_{\beta'} &= h_1 \eta_{\beta'} + (h_2 d_\gamma + h_3 V^\gamma + h_4 \epsilon^{\alpha\beta\gamma} d_\alpha V^\beta) (T^\gamma \eta)_{\beta'} \quad , \\ \hat{\phi}_{\beta'}^\dagger &= k_1 \eta_{\beta'}^\dagger + (k_2 d_\gamma + k_3 V^\gamma + k_4 \epsilon^{\alpha\beta\gamma} d_\alpha V^\beta) (\eta^\dagger T^\gamma)_{\beta'} \quad , \\ \hat{\Phi}_{n\beta} &= f_1(n) d_\beta + f_2(n) V^\beta + f_3(n) \epsilon^{\beta\gamma\delta} d_\gamma V^\delta \quad , \end{aligned} \quad (139)$$

where $V^\alpha \equiv \eta^\dagger T^\alpha \eta$ and h_i, k_i and $f_i(n)$ are functions which are made of $\eta_{\alpha'}, \eta_{\alpha'}^\dagger$ and d_α .³¹ The eigenvalue equation for $(\hat{\phi}, \hat{\phi}^\dagger, \hat{\Phi})$, $\det(\mathcal{M}_1 - \lambda I) = 0$, can be rewritten by that for $(h_i, k_i, f_j(n))$. The eigenvalues are obtained from the zeros of the determinant of the following matrix \mathcal{M}_2 .

	h_1	h_2	h_3	h_4	k_1	k_2	k_3	k_4	$f_1(n)$	$f_2(n)$	$f_3(n)$
$\eta_{\alpha'}$ $d_\alpha (T^\alpha \eta)_{\alpha'}$ $V_\alpha (T^\alpha \eta)_{\alpha'}$ $\epsilon_{\alpha\beta\gamma} d_\alpha V^\beta (T^\gamma \eta)_{\alpha'}$	a				c				C ₁		
$\eta_{\alpha'}^\dagger$ $d_\alpha (\eta^\dagger T^\alpha)_{\alpha'}$ $V_\alpha (\eta^\dagger T^\alpha)_{\alpha'}$ $\epsilon_{\alpha\beta\gamma} d_\alpha V^\beta (\eta^\dagger T^\gamma)_{\alpha'}$	d				b				C ₂		
d_α, m V_α, m $\epsilon_{\alpha\beta\gamma} d_\beta V^\gamma, m$	d ₁				d ₂				b ₁		

(140)

The components in each "box" are displayed in the following. For the purpose, we introduce here the following quantities which turn out to constitute the final result of the effective potential.

$$\begin{aligned} \text{4-dim scalar mass term: } S &= \eta^\dagger \eta \quad , \quad \text{D mass term: } d^2 = d_\alpha d_\alpha \quad , \\ \text{3-body term: } d \cdot V &= d_\alpha V^\alpha \quad , \quad \text{4-dim 4-body term: } V^2 = V^\alpha V_\alpha \quad , \\ &\quad \text{where } V^\alpha = \eta^\dagger T^\alpha \eta \quad . \end{aligned} \quad (141)$$

The 9 matrices in (140) are given by as follows.

³¹Their physical dimensions are as follows: $([h_1], [h_2], [h_3], [h_4]) = ([k_1], [k_2], [k_3], [k_4]) = (M^0, M^{-5/2}, M^{-2}, M^{-9/2}), ([f_1], [f_2], [f_3]) = (M^{-3/2}, M^{-1}, M^{-7/2})$.

The first row equation of (138), $\mathcal{M}_{\phi_{\alpha'}^\dagger \phi_{\beta'}} \hat{\phi}_{\beta'} + \mathcal{M}_{\phi_{\alpha'}^\dagger \phi_{\beta'}^\dagger} \hat{\phi}_{\beta'}^\dagger + \mathcal{M}_{\phi_{\alpha'}^\dagger \Phi_{n\beta}} \hat{\Phi}_{n\beta} = \lambda \hat{\phi}_{\alpha'}$, gives three matrices a, c, c_1 as

$$\begin{aligned} \mathbf{a} &= \begin{pmatrix} -\lambda - k^2 & \frac{g}{4}d^2 & \frac{g}{4}d \cdot V & 0 \\ g & -\lambda - k^2 - \frac{g^2}{4}\delta(0)S & 0 & \frac{i}{2}(gd \cdot V + g^2\delta(0)V^2) \\ -g^2\delta(0) & 0 & -\lambda - k^2 - \frac{g^2}{4}\delta(0)S & -\frac{i}{2}(gd^2 + g^2\delta(0)d \cdot V) \\ 0 & -\frac{i}{2}g^2\delta(0) & \frac{ig}{2} & -\lambda - k^2 - \frac{g^2}{4}\delta(0)S \end{pmatrix}, \\ \mathbf{C} &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{g^2}{4}\delta(0)S & 0 & -\frac{i}{2}g^2\delta(0)V^2 \\ -g^2\delta(0) & 0 & -\frac{g^2}{4}\delta(0)S & \frac{i}{2}g^2\delta(0)d \cdot V \\ 0 & \frac{i}{2}g^2\delta(0) & 0 & -\frac{g^2}{4}\delta(0)S \end{pmatrix}, \\ \mathbf{C}_1 &= \frac{1}{l\sqrt{l}} \begin{pmatrix} 0 & 0 & 0 \\ -g\pi n & 0 & 0 \\ 0 & -g\pi n & 0 \\ 0 & 0 & -g\pi n \end{pmatrix} \quad (142) \end{aligned}$$

The second row equation of (138), $\mathcal{M}_{\phi_{\alpha'} \phi_{\beta'}} \hat{\phi}_{\beta'} + \mathcal{M}_{\phi_{\alpha'} \phi_{\beta'}^\dagger} \hat{\phi}_{\beta'}^\dagger + \mathcal{M}_{\phi_{\alpha'} \Phi_{n\beta}} \hat{\Phi}_{n\beta} = \lambda \hat{\phi}_{\alpha'}^\dagger$, gives three matrices d, b, c_2 as

$$\begin{aligned} \mathbf{d} &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{g^2}{4}\delta(0)S & 0 & \frac{i}{2}g^2\delta(0)V^2 \\ -g^2\delta(0) & 0 & -\frac{g^2}{4}\delta(0)S & -\frac{i}{2}g^2\delta(0)d \cdot V \\ 0 & -\frac{i}{2}g^2\delta(0) & 0 & -\frac{g^2}{4}\delta(0)S \end{pmatrix}, \\ \mathbf{b} &= \begin{pmatrix} -\lambda - k^2 & \frac{g}{4}d^2 & \frac{g}{4}d \cdot V & 0 \\ g & -\lambda - k^2 - \frac{g^2}{4}\delta(0)S & 0 & -\frac{i}{2}(gd \cdot V + g^2\delta(0)V^2) \\ -g^2\delta(0) & 0 & -\lambda - k^2 - \frac{g^2}{4}\delta(0)S & \frac{i}{2}(gd^2 + g^2\delta(0)d \cdot V) \\ 0 & \frac{i}{2}g^2\delta(0) & -\frac{ig}{2} & -\lambda - k^2 - \frac{g^2}{4}\delta(0)S \end{pmatrix}, \\ \mathbf{C}_2 &= \frac{1}{l\sqrt{l}} \begin{pmatrix} 0 & 0 & 0 \\ -g\pi n & 0 & 0 \\ 0 & -g\pi n & 0 \\ 0 & 0 & -g\pi n \end{pmatrix} \quad (143) \end{aligned}$$

We note the relations $a = b^*, c = d^*, c_1 = c_2$.

The third row equation of (138), $\mathcal{M}_{\Phi_{m\alpha} \phi_{\beta'}} \hat{\phi}_{\beta'} + \mathcal{M}_{\Phi_{m\alpha} \phi_{\beta'}^\dagger} \hat{\phi}_{\beta'}^\dagger + \mathcal{M}_{\Phi_{m\alpha} \Phi_{n\beta}} \hat{\Phi}_{n\beta} = \lambda \hat{\Phi}_{m\alpha}$, gives three matrices d_1, d_2, b_1 as

$$\mathbf{d}_1 = \frac{1}{l\sqrt{l}} \begin{pmatrix} 0 & -\frac{g}{4}\pi m S & 0 & \frac{i}{2}\pi m V^2 \\ -g\pi m & 0 & -\frac{g}{4}\pi m S & -\frac{i}{2}\pi m d \cdot V \\ 0 & -\frac{g}{2}\pi m & 0 & -\frac{g}{4}\pi m S \end{pmatrix},$$

$$\mathbf{d}_2 = \frac{1}{l\sqrt{l}} \begin{pmatrix} 0 & -\frac{g}{4}\pi m S & 0 & -i\frac{g}{2}\pi m V^2 \\ -g\pi m & 0 & -\frac{g}{4}\pi m S & i\frac{g}{2}\pi m d \cdot V \\ 0 & i\frac{g}{2}\pi m & 0 & -\frac{g}{4}\pi m S \end{pmatrix},$$

$$\mathbf{b}_1 = \begin{pmatrix} (-\lambda - k^2 - (\frac{\pi m}{l})^2)^2 \delta_{mn} & 0 & 0 \\ 0 & (-\lambda - k^2 - (\frac{\pi m}{l})^2)^2 \delta_{mn} & 0 \\ 0 & 0 & (-\lambda - k^2 - (\frac{\pi m}{l})^2)^2 \delta_{mn} \end{pmatrix} \quad (144)$$

We note $d_1 = d_2^*$.

Using the formula (125), the determinant of \mathcal{M}_2 (140) decomposes as follows.

$$A \equiv \begin{pmatrix} a & c \\ d & b \end{pmatrix}, \quad \mathcal{M}_2 = \begin{pmatrix} A & \begin{matrix} c_1 \\ c_2 \end{matrix} \\ \begin{matrix} d_1 & d_2 \end{matrix} & b_1 \end{pmatrix},$$

$$\det(\mathcal{M}_1 - \lambda I) \sim \det \mathcal{M}_2 = \det \left(A - \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} b_1^{-1} \begin{pmatrix} d_1 & d_2 \end{pmatrix} \right) \times \det b_1 \quad (145)$$

The last expression is a product of two determinants. The eigenvalues from the right determinant $\det b_1 = 0$ gives

$$\lambda_m = -k^2 - \left(\frac{m\pi}{l}\right)^2, \quad m = 1, 2, 3, \dots, \quad (146)$$

which correspond to the massive KK-modes of the bulk scalar Φ . As for the left determinant, the matrix in the inside can be evaluated using the explicit expressions of (142), (143) and (144). Here we find a *smoothing* procedure of the singular term takes place as follows. We write the matrix A in (145) as $A(\delta(0))$ to show the $\delta(0)$ dependence explicitly. Then we find the following renormalization-like relation with respect to the singular quantity $\delta(0)$.

$$A(\delta(0)) - \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} b_1^{-1} \begin{pmatrix} d_1 & d_2 \end{pmatrix} = A(\delta(0)|_{sm}),$$

$$\delta(0)|_{sm} = \delta(0) + \frac{1}{l} \sum_{m=1}^{\infty} \frac{(\pi m/l)^2}{-\lambda - k^2 - (\pi m/l)^2}$$

$$= \delta(0) + \frac{1}{2l} \left(- \sum_{m \in \mathbf{Z}} 1 + \sum_{m \in \mathbf{Z}} \frac{\lambda + k^2}{\lambda + k^2 + (\pi m/l)^2} \right). \quad (147)$$

Using the relation $\sum_{m \in \mathbf{Z}} 1 = 2l\delta(0)$, $\delta(0)|_{sm}$ becomes a finite (regular) quantity.

$$\delta(0)|_{sm} = \frac{1}{2l} \sum_{m \in \mathbf{Z}} \frac{\lambda + k^2}{\lambda + k^2 + (\pi m/l)^2} = \begin{cases} \frac{1}{2} \sqrt{\lambda + k^2} \coth\{l\sqrt{\lambda + k^2}\} & \lambda > -k^2 \\ \frac{1}{2} \sqrt{-\lambda - k^2} \cot\{l\sqrt{-\lambda - k^2}\} & -k^2 > \lambda \end{cases} \quad (148)$$

The above relation manifestly shows that *the tower of the massive KK-modes smoothes the $\delta(0)$ singularity* appearing in the boundary part of the mass matrix \mathcal{M} , (134). (In the perturbative analysis of Sec.5, the present smoothing phenomenon corresponds to the cancellation of singularity appearing in the equations (49-54).) In the limit $|\lambda + k^2| \rightarrow \infty$, $\delta(0)|_{sm}$ reduces to $\delta(0)$.

Next we evaluate $\det A'$ ($A' \equiv A(\delta(0)|_{sm})$) in order to find remaining 8 eigenvalues. We repeat the formula: $\det A' = \det(a' - c'(b')^{-1}d') \det b'$, where the primed quantities are defined by those ones which are obtained by replacing $\delta(0)$, in matrices $A = (a, c / d, b)$, by $\delta(0)|_{sm}$. Now we may deal with 4×4 matrices $a' - c'(b')^{-1}d'$ and b' . We can explicitly calculate $\det A'$ (using an algebraic soft) and indeed obtain the expression. $\det A'$ is the function composed of the background quantities: $S, d^2, d \cdot V$ and V^2 defined in (141). It is better to see some "sections" rather than the full result in order to see the structure of the effective potential.

(i) $d_\alpha = 0$ ($d^2 = 0, d \cdot V = 0$)

This case gives the normalization value of the effective potential in the SUSY boundary condition.

$$\begin{aligned} \det A' &= (\lambda + k^2)^5 \left\{ \lambda + k^2 + \frac{g^2}{2} \delta(0)|_{sm} S \right\}^3, \\ \lambda_{1-5} &= -k^2 \text{ (5-fold)}, \\ \lambda_6, \lambda_7, \lambda_8 \text{ (3-fold)} : \quad \lambda + k^2 + \frac{g^2}{4} S \sqrt{\lambda + k^2} \coth l \sqrt{\lambda + k^2} &= 0. \end{aligned} \quad (149)$$

Let us look at the above full result from the perturbative approach and relate it to the result of Sec.5. First we do the propagator ($1/k^2$) expansion because the perturbative approach is based on the expansion around the free theory: $g = 0$.

$$\lambda + k^2 + \frac{g^2}{4} S \sqrt{k^2} \left\{ \coth l \sqrt{k^2} + \frac{\lambda}{k^2} \left(\frac{1}{2} \coth l \sqrt{k^2} - \frac{l \sqrt{k^2}}{2(\coth l \sqrt{k^2})^2} \right) + O\left(\frac{1}{(k^2)^2}\right) \right\} = 0 \quad (150)$$

Secondly we restrict the coupling and the considered configuration as follows.

$$\frac{g^2}{l} = \text{fixed} \ll 1, \quad l \sqrt{k^2} \leq 1. \quad (151)$$

The second equation is required for the validity of $1/k^2$ expansion and it says the 4d momentum integral should have the UV cutoff $1/l$. Taking into account the perturbative order up to the 1st order w.r.t. g^2/l and the 0-th order w.r.t. $1/k^2$, we obtain

$$\lambda = -k^2 \left(1 + \frac{g^2}{4} S \frac{\sqrt{k^2} \coth l \sqrt{k^2}}{k^2} \right). \quad (152)$$

This eigenvalue is consistent with the first part of (54). We must pick up one eigen value from $\lambda_1 - \lambda_5$, and three ones λ_6, λ_7 and λ_8 (3-fold) in order to be consistent with the perturbative result.

(ii) $V^2 \neq 0$, Others=0 ($d^2 = 0, d \cdot V = 0, S = 0$)

We examine the part that is composed of purely the 4-body interaction term operator V^2 .

$$\det A' = (\lambda + k^2)^8. \quad (153)$$

The term V^2 does not appear. This is desirable from the renormalization point of view. The absence of the 4-body interaction term in the SUSY normalization part implies the renormalization of this term works well without SUSY.

(iii) $d^2 \neq 0$, Others = 0 ($d \cdot V = 0, V^2 = 0, S = 0$) [equivalently $\eta = \eta^\dagger = 0$] This is a special case of (A), the decoupled case.

$$\begin{aligned} \det A' &= ((\lambda + k^2)^2 - \frac{g^2}{4}d^2)^4 \quad , \\ \lambda_{\pm} &= -k^2 \pm \frac{g}{2}\sqrt{d^2} \quad . \end{aligned} \quad (154)$$

Both λ_+ and λ_- are 4-fold eigenvalue. We pick up two eigen values of λ_+ (2-fold) and another two ones λ_- (2-fold). This result is consistent with Case (A).

(iv) $d \cdot V \neq 0$, Others=0 ($S = 0, d^2 = 0, V^2 = 0$)

We examine the part that is composed of purely the 3-body interaction operator $d \cdot V$.

$$\begin{aligned} \det A' &= (\lambda + k^2)^6 \left\{ (\lambda + k^2)^2 - \frac{g^3}{2} \delta(0) |_{sm} d \cdot V \right\} \quad , \\ \lambda_{1-6} &= -k^2 \text{ (6-fold)}, \\ \lambda_7, \lambda_8 : \quad & (\lambda + k^2)^2 - \frac{g^3}{2} d \cdot V \frac{\sqrt{\lambda + k^2}}{2} \coth l \sqrt{\lambda + k^2} = 0 \quad . \end{aligned} \quad (155)$$

The perturbative values are obtained as in (i). $1/k^2$ -expansion gives,

$$(\lambda + k^2)^2 - \frac{g^3}{4} d \cdot V \sqrt{k^2} \left\{ \coth l \sqrt{k^2} + \frac{\lambda}{k^2} \left(\frac{1}{2} \coth l \sqrt{k^2} - \frac{l \sqrt{k^2}}{2(\coth l \sqrt{k^2})^2} \right) + O\left(\frac{1}{(k^2)^2}\right) \right\} = 0 \quad (156)$$

Taking the terms up to the 0-th order w.r.t. $1/k^2$ and up to the 1-st order w.r.t. g^2/l , we obtain

$$(\lambda + k^2)^2 - \frac{g^3}{4} d \cdot V \sqrt{k^2} \coth l \sqrt{k^2} = 0 \quad . \quad (157)$$

This is a quadratic equation w.r.t. λ . The two roots λ_7, λ_8 satisfy

$$\lambda_7 \lambda_8 = (k^2)^2 \left(1 - \frac{g^3}{4} d \cdot V \frac{\sqrt{k^2} \coth l \sqrt{k^2}}{(k^2)^2} \right) \quad . \quad (158)$$

This is consistent with (54). As for the four eigenvalues, we pick up λ_1, λ_2 (2-fold) and λ_7, λ_8 .

(v) $S \neq 0$, Others = 0 ($d^2 = 0, d \cdot V = 0, V^2 = 0$)

We examine the part that is composed of purely the mass term operator $S = \eta^\dagger \eta$. The form of $\det A'$ is the same as the case (i). Hence the effective potential is the

same as (i). The 4D scalar mass term appears in the intermediate procedure, but it disappears in the SUSY boundary condition. This shows the renormalization about the scalar mass term works with the help of SUSY.

11 Appendix C: Background Fields and On-Shell Condition

We show the background fields taken in Sec.6 satisfy the field equation of (40), the on-shell condition, for a special case given below. The assumed forms are

$$\begin{aligned} \varphi_\alpha(x^5) &= \bar{\varphi}_\alpha \epsilon(x^5) \quad , \quad a_{5\alpha}(x^5) = \bar{a}_\alpha \epsilon(x^5) \quad , \\ \eta_{\alpha'} &= \text{const} \quad , \quad \eta_{\alpha'}^\dagger = \text{const} \quad , \quad d_\alpha = \chi_\alpha^3 - \partial_5 \varphi_\alpha + g(a_5 \times \varphi)_\alpha = \text{const} \end{aligned} \quad (159)$$

where $\epsilon(x)$ is the periodic sign function defined by (62). First we stress that the total derivative terms, appearing in the derivation of the field equation (40), can be safely put to 0 because of the periodicity property. Using the relation (75) and the condition $m_{\alpha'\beta'} = \lambda_{\alpha'\beta'\gamma'} = 0$, the equations in (40) can be expressed as

$$\begin{aligned} 2\bar{\varphi}_\alpha \partial_5(\delta(x^5) - \delta(x^5 - l)) - g^2 \epsilon(x^5)((\bar{a} \times \bar{\varphi}) \times \bar{a})_\alpha + g \partial_5 \delta(x^5) \cdot \eta^\dagger T^\alpha \eta + g \partial_5 \delta(x^5 - l) \cdot \eta'^\dagger T^\alpha \eta' \\ + g^2 [(\delta(x^5) \eta^\dagger T \eta + \delta(x^5 - l) \eta'^\dagger T \eta') \times \bar{a}]_\alpha \epsilon(x^5) = 0 \quad , \\ 2\bar{a} \partial_5(\delta(x^5) - \delta(x^5 - l)) - g^2 \epsilon(x^5)(\bar{\varphi} \times (\bar{a} \times \bar{\varphi}))_\alpha \\ - g^2 [(\delta(x^5) \eta^\dagger T \eta + \delta(x^5 - l) \eta'^\dagger T \eta') \times \bar{\varphi}]_\alpha \epsilon(x^5) = 0 \quad , \\ \chi_\alpha^3 + g(\delta(x^5) \eta^\dagger T^\alpha \eta + \delta(x^5 - l) \eta'^\dagger T^\alpha \eta') = 0 \quad , \\ g\{\chi_\beta^3 - 2\bar{\varphi}_\beta(\delta(x^5) - \delta(x^5 - l)) + g(\bar{a} \times \bar{\varphi})_\beta\}(T^\beta \eta)_{\alpha'} = 0 \quad , \\ g\{\chi_\beta^3 - 2\bar{\varphi}_\beta(\delta(x^5) - \delta(x^5 - l)) + g(\bar{a} \times \bar{\varphi})_\beta\}(T^\beta \eta')_{\alpha'} = 0 \end{aligned} \quad (160)$$

We note the following things.

1. When $\bar{a}_\alpha \propto \bar{\varphi}_\alpha$, the following relations hold: $(\bar{a} \times \bar{\varphi})_\alpha = f_{\alpha\beta\gamma} \bar{a}_\beta \bar{\varphi}_\gamma = 0$.
2. $\partial_5(\delta(x^5) - \delta(x^5 - l)) \times \text{const} = 0$ with the Neumann boundary condition: $\partial_5(\delta A_\alpha^5)|_{x^5=0} = \partial_5(\delta A_\alpha^5)|_{x^5=l} = 0$.
3. $\epsilon(x^5)^2 = 1, \epsilon(x^5)^3 = \epsilon(x^5), \partial_5(\epsilon(x^5)) = 2(\delta(x^5) - \delta(x^5 - l)), \frac{1}{2}\partial_5\{\epsilon(x^5)^2\} = (\delta(x^5) - \delta(x^5 - l))\epsilon(x^5) = 0$.

Then we can conclude that (159) is a solution of the field equation (40) for the following choice.

$$\begin{aligned} \text{const} \times \bar{a}_\alpha &= \bar{\varphi}_\alpha = -\frac{g}{2} \eta^\dagger T^\alpha \eta = \frac{g}{2} \eta'^\dagger T^\alpha \eta' \quad , \\ \chi_\alpha^3 &= -g(\delta(x^5) - \delta(x^5 - l)) \eta^\dagger T^\alpha \eta \quad . \end{aligned} \quad (161)$$

In this choice $d_\alpha = 0$ is concluded. The more general solution is given in [38].

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